

# Applications of Propositional Logic

Section 1.2

# Applications of Propositional Logic: Summary

- Translating English to Propositional Logic
- System Specifications
- Boolean Searching
- Logic Puzzles
- Logic Circuits

# Translating English Sentences

- Steps to convert an English sentence to a statement in propositional logic
  - Identify atomic propositions and represent using propositional variables.
  - Determine appropriate logical connectives
- “If I go to Harry’s or to the country, I will not go shopping.”
  - $p$ : I go to Harry’s
  - $q$ : I go to the country.
  - $r$ : I will go shopping.

If  $p$  or  $q$  then not  $r$ .

$$(p \vee q) \rightarrow \neg r$$

# Example

**Problem:** Translate the following sentence into propositional logic:

“You can access the Internet from campus only if you are a computer science major or you are not a freshman.”

**One Solution:** Let  $a$ ,  $c$ , and  $f$  represent respectively “You can access the internet from campus,” “You are a computer science major,” and “You are a freshman.”

$$a \rightarrow (c \vee \neg f)$$

# System Specifications

- System and Software engineers take requirements in English and express them in a precise specification language based on logic.

**Example:** Express in propositional logic:

“The automated reply cannot be sent when the file system is full”

**Solution:** One possible solution: Let  $p$  denote “The automated reply can be sent” and  $q$  denote “The file system is full.”

$$q \rightarrow \neg p$$

# Consistent System Specifications

**Definition:** A list of propositions is *consistent* if it is possible to assign truth values to the proposition variables so that each proposition is true.

**Exercise:** Are these specifications consistent?

- “The diagnostic message is stored in the buffer or it is retransmitted.”
- “The diagnostic message is not stored in the buffer.”
- “If the diagnostic message is stored in the buffer, then it is retransmitted.”

**Solution:** Let  $p$  denote “The diagnostic message is stored in the buffer.” Let  $q$  denote “The diagnostic message is retransmitted” The specification can be written as:  $p \vee q, p \rightarrow q, \neg p$ . When  $p$  is false and  $q$  is true all three statements are true. So the specification is consistent.

- What if “The diagnostic message is not retransmitted” is added.

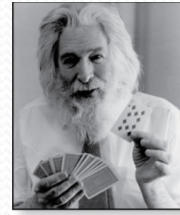
**Solution:** Now we are adding  $\neg q$  and there is no satisfying assignment. So the specification is not consistent.

# Consistent System Specifications

- “The diagnostic message is stored in the buffer or it is retransmitted.”
- “The diagnostic message is not stored in the buffer.”
- “If the diagnostic message is stored in the buffer, then it is retransmitted.”
- What if “The diagnostic message is not retransmitted” is added.

$p$	$q$	$p \vee q$	$\neg p$	$p \rightarrow q$	$\neg q$
F	F	F	T	T	T
F	T	T	T	T	F
T	F	T	F	F	T
T	T	T	F	T	F

# Logic Puzzles



Raymond  
Smullyan  
(Born 1919)

- An island has two kinds of inhabitants, *knights*, who always tell the truth, and *knaves*, who always lie.
- You go to the island and meet A and B.
  - A says “B is a knight.”
  - B says “The two of us are of opposite types.”

**Example:** What are the types of A and B?



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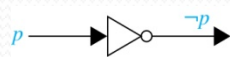
**Solution:** Let  $p$  and  $q$  be the statements that A is a knight and B is a knight, respectively. So, then  $\neg p$  represents the proposition that A is a knave and  $\neg q$  that B is a knave.

- If A is a knight, then  $p$  is true. Since knights tell the truth,  $q$  must also be true. Then  $(p \wedge \neg q) \vee (\neg p \wedge q)$  would have to be true, but it is not. So, A is not a knight and therefore  $\neg p$  must be true.
- If A is a knave, then B must not be a knight since knaves always lie. So, then both  $\neg p$  and  $\neg q$  hold since both are knaves.

# Logic Circuits

## (Studied in depth in Chapter 12)

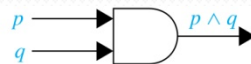
- Electronic circuits; each input/output signal can be viewed as a 0 or 1.
  - 0 represents **False**
  - 1 represents **True**
- Complicated circuits are constructed from three basic circuits called gates.



Inverter



OR gate



AND gate

- The inverter (**NOT gate**) takes an input bit and produces the negation of that bit.
  - The **OR gate** takes two input bits and produces the value equivalent to the disjunction of the two bits.
  - The **AND gate** takes two input bits and produces the value equivalent to the conjunction of the two bits.
- More complicated digital circuits can be constructed by combining these basic circuits to produce the desired output given the input signals by building a circuit for each piece of the output expression and then combining them. For example:

