Homework 6

Section 2.3

4. Find the domain and range of these functions. Note that in each case, to find the domain, determine the set of elements assigned values by the function.
   (a) the function that assigns to each nonnegative integer its last digit
   (b) the function that assigns the next largest integer to a positive integer
   (c) the function that assigns to a bit string the number of one bits in the string (for example, $f(\text{“101100”}) = 3$, $f(\text{“000010”}) = 1$)
   (d) the function that assigns to a bit string the number of bits in the string (for example, $f(\text{“101100”}) = 6$, $f(\text{“00”}) = 2$)

12. Determine whether each of these functions from Z to Z is one-to-one.
   (a) $f(n) = n - 1$
   (b) $f(n) = n^2 + 1$
   (c) $f(n) = n^3$
   (d) $f(n) = \lceil \frac{n}{2} \rceil$

22. Determine whether each of these functions is a bijection from R to R.
   (a) $f(x) = -3x + 4$
   (b) $f(x) = -3x^2 + 7$
   (c) $f(x) = \frac{x+1}{x+2}$
   (d) $f(x) = x^5 + 1$

30. Let $S = \{1, 0, 2, 4, 7\}$. Find $f(S)$ if
   (a) $f(x) = 1$
   (b) $f(x) = 2x + 1$
   (c) $f(x) = \lceil \frac{x}{5} \rceil$
   (d) $f(x) = \lfloor \frac{x^2+1}{3} \rfloor$

36. Find $f \circ g$ and $g \circ f$, where $f(x) = x^2 + 1$ and $g(x) = x + 2$, are functions from R to R.

Section 2.4

2. What is the term a_8 of the sequence $\{a_n\}$ if a_n equals
   (a) 2^{n-1}
   (b) 7
   (c) $1+(-1)^n$
   (d) $-(-2)^n$

10. Find the first six terms of the sequence defined by each of these recurrence relations and initial conditions.
    (a) $a_n = -2a_{n-1}$, where $a_0 = -1$
    (b) $a_n = a_{n-1} - a_{n-2}$, where $a_0 = 2$, $a_1 = -1$
    (c) $a_n = 3a_{n-1}^2$, where $a_0 = 1$
    (d) $a_n = na_{n-1} + a_{n-2}^2$, where $a_0 = -1$, $a_1 = 0$
    (e) $a_n = a_{n-1} - a_{n-2} + a_{n-3}$, where $a_0 = 1$, $a_1 = 1$, $a_2 = 2$

16. Find the solution to each of these recurrence relations with the given initial conditions. Use an iterative approach such as that used in Example 10.
    (a) $a_n = -a_{n-1}$, where $a_0 = 5$
    (b) $a_n = a_{n-1} + 3$, where $a_0 = 1$
    (c) $a_n = a_{n-1} - n$, where $a_0 = 4$
    (d) $a_n = 2a_{n-1} - 3$, where $a_0 = -1$

26. For each of these lists of integers, provide a simple formula or rule that generates the terms of an integer sequence that begins with the given list. Assuming that your formula or rule is correct, determine the next three terms of the sequence.
    (a) $3, 6, 11, 18, 27, 38, 51, 66, 83, 102, \ldots$
    (b) $7, 11, 15, 19, 23, 27, 31, 35, 39, 43, \ldots$
    (c) $1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, \ldots$
    (d) $1, 2, 2, 2, 3, 3, 3, 3, 3, 5, 5, 5, 5, 5, 5, 5, \ldots$
    (e) $0, 2, 8, 26, 80, 242, 728, 2186, 6560, 19682, \ldots$

30. What are the values of these sums, where $S = \{1, 3, 5, 7\}$?
    (a) $\sum_{j\in S} j$
    (b) $\sum_{j\in S} \frac{1}{j}$
    (c) $\sum_{j\in S} 1$
    (d) $\sum_{i=0}^{4} i$
    (e) $\sum_{i=3}^{5} (i-1)^2$