Solve the task scheduling problem for the following tasks (labeled alphabetically) with the specified start and end times: A [12, 13], B [8, 11], C [1, 3], D [1, 2], E [3, 4], F [6, 10], G [7, 9], H [10, 12], I [4, 6], J [3, 7], K [2, 5], $\mathrm{L}[9,13], \mathrm{M}[3,6], \mathrm{N}[5,8]$

Solution: First, sort tasks by increasing start time. For tasks that have the same start time, we can choose arbitrarily among them. So, we process tasks in this order:

1. Task C: $[1,3]$
2. Task J: $[3,7]$
3. Task F: $[6,10]$
4. Task H: $[10,12]$
5. Task D: $[1,2]$
6. Task E: $[3,4]$
7. Task G: $[7,9]$
8. Task K: $[2,5]$
9. Task I: $[4,6]$
10. Task B: $[8,11]$
11. Task M: $[3,6]$
12. Task N: $[5,8]$
13. Task L: $[9,13]$
14. Task A: $[12,13]$

Schedule the first task (Task C: $[1,3]$ ) on a machine. We have no machines, so create one. Store with this machine the last end time (3).


Process the next task (Task D: $[1,2]$ ). Check if it conflicts on the machine with the earliest end time (machine 1). It does, so we create a new machine, schedule a task on it, and store with the machine the last end time (2).


Process the next task (Task K: $[2,5]$ ). Check if it conflicts on the machine with the earliest end time - that's machine 2 now. It doesn't, so we can schedule it on machine 2. Update machine 2's last end time to be the end time of task K.


Process Task M: [3, 6]. Check if it conflicts with the machine with the earliest end time - that's machine 1 now. It doesn't, so we can schedule it on machine 1. Update machine 1's last end time to be the end time of task M.


Process Task J: [3, 7]. Check if it conflicts with machine 2 (since that machine has the earliest end time). It does, so create a new machine.


Process Task E: [3, 4]. Check if it conflicts with machine 2 (since that machine has the earliest end time). It does, so create a new machine.


Process Task I: $[4,6]$. Check if it conflicts on machine 4 (since that machine has the earliest end time), which it doesn't. Schedule it there.


Process Task N: [5, 8]. Check if it conflicts with machine 2 (since that machine has the earliest end time), which it doesn't. Schedule it there.


Process Task F: [6, 10]. The machine with the earliest end time is machine 1 and machine 4 . They are equivalent for our purposes of scheduling task F , so we compare against only one of them - let's pick machine 1.


Process Task G: $[7,9]$. Check if it conflicts with machine 4 (the machine with the earliest end time), and it doesn't. Schedule it there.


Process Task B: $[8,11]$. Check if it conflicts with machine 3 (machine with earliest end time). It doesn't, so schedule it there.


Process Task L: [9, 13]. Check if it conflicts with machine 2 (machine with earliest end time). It doesn't, so schedule it there.


Process Task H: [10, 12]. Check if it conflicts with machine 4 (machine with earliest end time). It doesn't, so schedule it there.


Process Task A: [12, 13]. Check if it conflicts with machine 1 (machine with earliest end time). It doesn't, so schedule it there.


