Homework 5 (50 pts)

1. (10 points) Let $S = \{a, b, c, d, e, f, g\}$ be a collection of items with weight-benefit values as follows: $a(3, 12)$, $b(6, 12)$, $c(6, 9)$, $d(1, 5)$, $e(2, 5)$, $f(10, 10)$, $g(3, 9)$. For example, item a weighs 3 lbs and is worth a total of \$12. What is an optimal solution to the **fractional** knapsack problem for S assuming we have a knapsack that can hold a total of 11 lbs? Show your work.

2. (10 points) Suppose we are given a set of tasks specified by pairs of start times and finish times as $T = \{(5, 6), (3, 7), (1, 2), (6, 8), (1, 3), (7, 9), (1, 4), (2, 5), (4, 9), (7, 10)\}$. Solve the task scheduling problem for these tasks.

3. Consider the single machine scheduling problem where we are given a set T of tasks specified by their start times and finish times, as in the task scheduling problem, except now we have only one machine and we wish to maximize the number of tasks that this single machine performs.
   (a) (7 points) Design a greedy algorithm for this single machine scheduling problem. What is the running time of your algorithm?
   (b) (3 points) Prove the correctness of your algorithm.

4. (10 points) For each of the following recurrence equations which describe the running time $T(n)$ of a recursive algorithm, use the master method to express the asymptotic complexity (assuming that $T(n) = c$ for $n < d$, for constants $c > 0$ and $d \geq 1$).
   (a) $T(n) = 2T(n/2) + \log n$
   (b) $T(n) = 8T(n/2) + n^2$
   (c) $T(n) = 7T(n/3) + n$
   (d) $T(n) = 4T(n/2) + n^2$
   (e) $T(n) = 3T(n/2) + n^2$

5. (10 points) What is the best way to multiply a chain of matrices with dimensions that are 10×5, 5×2, 2×20, 20×12, 12×4, and 4×60? Show your work (including two tables - one indicating the index k which gives the final multiply index for each subproblem, and the second table which indicates the optimal number of multiplications for each subproblem).