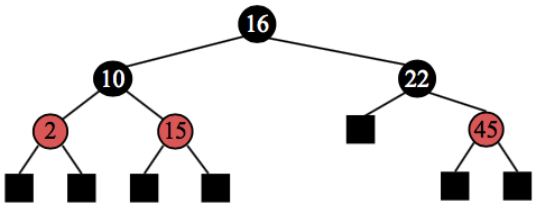
Homework 4 (55pts)

1. (5 points) Consider the following sequence of keys: (18, 30, 50, 12, 1). Insert the items with this set of keys in the order given into the red-black tree in the figure below. Draw the tree after **each** insertion.



2. (10 points) Design and give the **pseudocode** for an $O(\log n)$ algorithm that determines whether a red-black tree with n keys stores any keys within a certain (closed) interval. That is, the input to the algorithm is a red-black tree T and two keys, l and r, where $l \leq r$. If T has at least one key k such that $l \leq k \leq r$, then the algorithm returns true, otherwise it returns false. *Hint:* You can use the TreeSearch algorithm (page 146) as a subroutine.

3. (a) (5 points) Draw the merge-sort tree for an execution of the merge-sort algorithm on the input sequence: (2, 5, 16, 4, 10, 23, 39, 18, 26, 15) (like in Figure. 4.2).

   (b) (5 points) Draw the quick-sort tree for an execution of the quick-sort algorithm on the input sequence from part (a) (like in Figure 4.12). Use the last element as the pivot.

   (c) (3 points) What is the running time of the version of quick-sort that uses the element at rank $\lfloor \frac{n}{2} \rfloor$ as the pivot, provided that the input sequence is already sorted? Explain.

4. (10 points) Suppose we are given a sequence S of n elements, each of which is colored red or blue. Assuming S is represented by an array, give a linear-time **in-place** algorithm for ordering S so that all the blue elements are listed before all the red elements. What is the running time of your method?

5. (10 points) Let A and B be two sequences of n integers each. Give an integer m, describe an $O(n \log n)$ time algorithm for determining if there is an integer a in A and an integer b in B such that $m = a + b$.

6. (a) (5 points) Suppose we are given a sequence S of n elements, each of which is an integer in the range $[0, n^2 - 1]$. Describe a simple method for sorting S in $O(n)$ time. [*Hint*: think of alternative ways of viewing the elements].

   (b) (2 points) Does the running time of radix-sort depend on the order of keys in the input? Explain.