

This is given:

matrix	$A_0$	$A_1$	$A_2$	$A_3$	$A_4$	$A_5$
dimension	$30 \times 35$	$35 \times 15$	$15 \times 5$	$5 \times 10$	$10 \times 20$	$20 \times 25$

- $d_0$  30
- $d_1$  35
- $d_2$  15
- $d_3$  5
- $d_4$  10
- $d_5$  20
- $d_6$  25

N	0	1	2	3	4	5
0	0					
1		0				
2			0			
3				0		
4					0	
5						0

Step 1. If there's only one matrix, it costs nothing to multiply since it is already provided.

- $A_0 A_1$   $30 \cdot 35 \cdot 15 = 15750$
- $A_1 A_2$   $35 \cdot 15 \cdot 5 = 2625$
- $A_2 A_3$   $15 \cdot 5 \cdot 10 = 750$
- $A_3 A_4$   $5 \cdot 10 \cdot 20 = 1000$
- $A_4 A_5$   $10 \cdot 20 \cdot 25 = 5000$

Step 2. Multiplying two matrices together

N	0	1	2	3	4	5
0	0	15750				
1		0	2625			
2			0	750		
3				0	1000	
4					0	5000
5						0

K	0	1	2	3	4	5
0		0				
1			1			
2				2		
3					3	
4						4
5						

$$A_0 A_1 A_2 = \min \begin{cases} [A_0 A_1] A_2 & 0 + 2625 + 30 \cdot 35 \cdot 5 = 7875 & k=0 \\ [A_0 A_2] A_1 & 15750 + 0 + 30 \cdot 15 \cdot 5 = 18000 & k=1 \end{cases}$$

Step 3. Multiplying three matrices together

N	0	1	2	3	4	5
0	0	15750	7875			
1		0	2625			
2			0	750		
3				0	1000	
4					0	5000
5						0

K	0	1	2	3	4	5
0		0	0			
1			1			
2				2		
3					3	
4						4
5						

Note we record  $k[0][2]=0$  because (highlighted in green above), the minimum number that we recorded in  $N[0][2]$  came when we did the last multiplication at matrix  $A_0$ .

$$A_1 A_2 A_3 = \min \begin{cases} A_1 (A_2 A_3) & 0 + 750 + 35 \cdot 15 \cdot 10 = 6000 & k=1 \\ (A_1 A_2) A_3 & 2625 + 0 + 35 \cdot 5 \cdot 10 = 4375 & k=2 \end{cases}$$

N	0	1	2	3	4	5
0	0	15750	7875			
1		0	2625	4375		
2			0	750		
3				0	1000	
4					0	5000
5						0

K	0	1	2	3	4	5
0		0	0			
1			1	2		
2				2		
3					3	
4						4
5						

$k[1][2]=2$  because (highlighted in green above), the minimum number that we recorded in  $N[1][3]$  came when we did the last multiplication at matrix  $A_0$ .

$$\begin{array}{l}
 i=2 \\
 j=4
 \end{array}
 A_2 A_3 A_4 \quad \min \left\{ \begin{array}{l} (A_2)(A_3 A_4) \\ (A_2 A_3)(A_4) \end{array} \right. \quad \begin{array}{l} 0 + 1000 + 15 \cdot 5 \cdot 20 = 2500 \quad k=2 \\ 750 + 0 + 15 \cdot 10 \cdot 20 = 3750 \quad k=3 \end{array}$$

$$\begin{array}{l}
 i=3 \\
 j=5
 \end{array}
 A_3 A_4 A_5 \quad \min \left\{ \begin{array}{l} (A_3)(A_4 A_5) \\ (A_3 A_4)(A_5) \end{array} \right. \quad \begin{array}{l} 0 + 5000 + 5 \cdot 10 \cdot 25 = 6250 \quad k=3 \\ 1000 + 0 + 5 \cdot 20 \cdot 25 = 3500 \quad k=4 \end{array}$$

N	0	1	2	3	4	5
0	0	15750	7875			
1		0	2625	4375		
2			0	750	2500	
3				0	1000	3500
4					0	5000
5						0

K	0	1	2	3	4	5
0		0	0			
1			1	2		
2				2	2	
3					3	4
4						4
5						

$$\begin{array}{l}
 i=0 \\
 j=3
 \end{array}
 A_0 A_1 A_2 A_3 \quad \min \left\{ \begin{array}{l} (A_0 A_1 A_2 A_3) \\ (A_0 A_1 A_2) A_3 \\ (A_0 A_1 A_3) A_2 \end{array} \right. \quad \begin{array}{l} 0 + 4375 + 30 \cdot 35 \cdot 10 = 14875 \quad k=0 \\ 15750 + 750 + 30 \cdot 15 \cdot 10 = 21000 \quad k=1 \\ 7875 + 0 + 30 \cdot 5 \cdot 10 = 9375 \quad k=2 \end{array}$$

$$\begin{array}{l}
 i=1 \\
 j=4
 \end{array}
 A_1 A_2 A_3 A_4 \quad \min \left\{ \begin{array}{l} (A_1 A_2 A_3 A_4) \\ (A_1 A_2 A_3) A_4 \\ (A_1 A_2 A_4) A_3 \end{array} \right. \quad \begin{array}{l} 0 + 2500 + 35 \cdot 15 \cdot 20 = 13000 \quad k=1 \\ 2625 + 1000 + 35 \cdot 5 \cdot 20 = 7125 \quad k=2 \\ 4375 + 0 + 35 \cdot 10 \cdot 20 = 11375 \quad k=3 \end{array}$$

$$\begin{array}{l}
 i=2 \\
 j=5
 \end{array}
 A_2 A_3 A_4 A_5 \quad \min \left\{ \begin{array}{l} A_2 A_3 A_4 A_5 \\ A_2 A_3 A_4 A_5 \\ A_2 A_3 A_4 A_5 \end{array} \right. \quad \begin{array}{l} 0 + 3500 + 15 \cdot 5 \cdot 25 = 5375 \quad k=2 \\ 750 + 5000 + 15 \cdot 10 \cdot 25 = 9500 \quad k=3 \\ 2500 + 0 + 15 \cdot 20 \cdot 25 = 10000 \quad k=4 \end{array}$$

Step 4. Multiplying four matrices together

N	0	1	2	3	4	5
0	0	15750	7875	9375		
1		0	2625	4375	7125	
2			0	750	2500	5375
3				0	1000	3500
4					0	5000
5						0

K	0	1	2	3	4	5
0		0	0	2		
1			1	2	2	
2				2	2	2
3					3	4
4						4
5						

$$\begin{array}{l}
 i=0 \\
 j=4
 \end{array}
 A_0 A_1 A_2 A_3 A_4 \quad \min \left\{ \begin{array}{l} A_0 A_1 A_2 A_3 A_4 \\ A_0 A_1 A_2 A_3 A_4 \\ A_0 A_1 A_2 A_3 A_4 \\ A_0 A_1 A_2 A_3 A_4 \end{array} \right. \quad \begin{array}{l} 0 + 7125 + 30 \cdot 35 \cdot 20 = 28125 \quad k=0 \\ 15750 + 2500 + 30 \cdot 15 \cdot 20 = 27250 \quad k=1 \\ 7875 + 1000 + 30 \cdot 5 \cdot 20 = 11875 \quad k=2 \\ 9375 + 0 + 30 \cdot 10 \cdot 20 = 15375 \quad k=3 \end{array}$$

$$\begin{array}{l}
 i=1 \\
 j=5
 \end{array}
 A_1 A_2 A_3 A_4 A_5 \quad \min \left\{ \begin{array}{l} A_1 A_2 A_3 A_4 A_5 \\ A_1 A_2 A_3 A_4 A_5 \\ A_1 A_2 A_3 A_4 A_5 \\ A_1 A_2 A_3 A_4 A_5 \end{array} \right. \quad \begin{array}{l} 0 + 5375 + 35 \cdot 15 \cdot 25 = 18500 \quad k=1 \\ 2625 + 3500 + 35 \cdot 5 \cdot 25 = 10500 \quad k=2 \\ 4375 + 5000 + 35 \cdot 10 \cdot 25 = 18125 \quad k=3 \\ 7125 + 0 + 35 \cdot 20 \cdot 25 = 24625 \quad k=4 \end{array}$$

Step 5. Multiplying five matrices together

N	0	1	2	3	4	5
0	0	15750	7875	9375	11875	
1		0	2625	4375	7125	10500
2			0	750	2500	5375
3				0	1000	3500
4					0	5000
5						0

K	0	1	2	3	4	5
0		0	0	2	2	
1			1	2	2	2
2				2	2	2
3					3	4
4						4
5						

$i=0$   
 $j=5$   
 min
 

$A_0 A_1 A_2 A_3 A_4 A_5$	$0 + 10500$	$+ 30 \cdot 35 \cdot 25 = 36750$	$k=0$
$A_0 A_1 A_2 A_3 A_4 A_5$	$15750 + 5375$	$+ 30 \cdot 15 \cdot 25 = 32375$	$k=1$
$A_0 A_1 A_2 A_3 A_4 A_5$	$7875 + 3500$	$+ 30 \cdot 5 \cdot 25 = 15125$	$k=2$
$A_0 A_1 A_2 A_3 A_4 A_5$	$9375 + 5000$	$+ 30 \cdot 10 \cdot 25 = 21875$	$k=3$
$A_0 A_1 A_2 A_3 A_4 A_5$	$11875 + 0$	$+ 30 \cdot 20 \cdot 25 = 26875$	$k=4$

Step 6. Multiplying six matrices together

N	0	1	2	3	4	5
0	0	15750	7875	9375	11875	15125
1		0	2625	4375	7125	10500
2			0	750	2500	5375
3				0	1000	3500
4					0	5000
5						0

K	0	1	2	3	4	5
0		0	0	2	2	2
1			1	2	2	2
2				2	2	2
3					3	4
4						4
5						

Finished building N table (left). It says that the minimum number of scalar operations it will take to multiply these 6 matrices together is 15125. The k table (right) explains how the chain of matrices was parenthesized which minimized the number of scalar operations. Interpreting the k-table gives the **solution**

$$((A_0) \times (A_1 \times A_2)) \times ((A_3 \times A_4) \times (A_5))$$

$k[0][5]$  determines the last multiplication, effectively splitting the chain of matrices into to large matrices from  $A_0 \dots A_2$  on the left and  $A_3 \dots A_5$  on the right.

$k[0][2]$  describes how to multiply the left chain (from  $A_0 \dots A_2$ )

$k[3][5]$  describes how to multiply the right chain (from  $A_3 \dots A_5$ )