Equivalent: Implication $p \to q$ and Contrapositive of $p \to q$; Converse of $p \to q$ and Inverse of $p \to q$

p	q	Negation $\neg q$	Conjunction $p \wedge q$	Disjunction (Inclusive OR) $p \vee q$	XOR (Exclusive Or) $p \oplus q$	Implication $p \to q$	Converse of $p \to q$: $q \to p$	Inverse of $p \to q$: $\neg p \to \neg q$	Contrapositive of $p \to q$: $\neg q \to \neg p$	Biconditional $p \leftrightarrow q$
T	T	F	T	T	F	T	T	T	T	T
T	F	T	F	T	T	F	T	T	F	F
F	T		F	T	T	T	F	F	T	F
F	F		F	F	F	T	T	T	T	T

2^n = # rows

Key Concepts:

- Two propositions are equivalent ($p \equiv q$) if they always have the same truth-value.
- A list of compound propositions is consistent if it's possible to assign truth-values to the atomic propositions such that each compound proposition in the list is true.
- A tautology is a proposition that's always true.
- A contradiction is a proposition that's always false.
- A contingency is a proposition that's neither a tautology nor a contradiction.
- A compound proposition is satisfiable if there is an assignment of truth values to its variables that make it true.
- A argument in propositional logic is a sequence of propositions. All but the final proposition are called premises. The last statement is the conclusion.
- An argument is valid if the premises imply the conclusion.
- An argument form is an argument that is valid no matter what propositions are substituted into its propositional variables.

Ways of expressing $p \to q$:

- If p, then q
- If p, q
- q unless ¬p
- q if p
- q whenever p
- q follows from p
- p implies q
- p only if q
- q when p
- p is sufficient for q
- q is necessary for p
- It is necessary to q to p
- A necessary condition for p is q
- A sufficient condition for q is p

Ways of expressing $p \leftrightarrow q$:

- *p* is necessary and sufficient for *q*
- If *p* then *q*, and conversely
- *p* iff *q*

Law		
De Morgan's	$\neg\exists x P(x) \equiv \forall x \neg P(x)$ $\neg\forall x P(x) \equiv \exists x \neg P(x)$	
	$\neg(p \wedge q) \equiv \neg p \vee \neg q$	$\neg(p \vee q) \equiv \neg p \wedge \neg q$
Double negation	$\neg(\neg p) \equiv p$	
Negation	$p \wedge \neg p \equiv \mathbf{F}$	$p \vee \neg p \equiv \mathbf{T}$
Identity	$p \wedge \mathbf{T} \equiv p$	$p \vee \mathbf{F} \equiv p$
Domination	$p \wedge \mathbf{F} \equiv \mathbf{F}$	$p \vee \mathbf{T} \equiv \mathbf{T}$
Idempotent	$p \wedge p \equiv p$	$p \vee p \equiv p$
Communtative	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
Associative	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
Distributive	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
Absorption	$p \wedge (p \vee q) \equiv p$	$p \vee (p \wedge q) \equiv p$
Implication	$p \to q \equiv \neg p \vee q$	
Contrapositive	$p \to q \equiv \neg q \to \neg p$	

Universal Quantifier: $\forall x P(x)$

- "For all x, P(x)"
- "For any arbitrary x, P(x)"
- "For every x, P(x)"
- "For each x, P(x)"

Existential Quantifier: $\exists x P(x)$

- "There exists an x such that P(x)"
- "There is an x such that P(x)"
- "For some x, P(x)"
- "There is at least one x such thatP(x)"

Rules of Inference

Name	Modus Ponens (MP)	Modus Tollens (MT)	Hypothetical Syllogism (HS)	Disjunctive Syllogism (DS)	Addition	Simplification	Conjunction	Resolution
Premises	$p \to q$ p	$p \to q$ $\neg q$	$p \to q$ $q \to r$	$p \lor q$ $\neg p$	p	$p \land q$	p q	$\neg p \lor r$ $p \lor q$
Conclusion	q	$\neg p$	$p \to r$	q	$p \lor q$	p	$p \land q$	$q \lor r$

Name	Universal Instantiation (UI)	Universal Generalization (UG)	Existential Instantiation (EI)	Existential Generalization (EG)	Universal Modus Ponens
Premises	$\forall x P(x)$	$P(c)$ for an <u>arbitrary</u> *c*	$\exists x P(x)$	$P(c)$ for some element *c*	$\forall x (P(x) \to Q(x))$ $P(a)$ where *a* is a particular element in the domain
Conclusion	$P(c)$	$\forall x P(x)$	$P(c)$ for some element *c*	$\exists x P(x)$	$Q(a)$

Proof methods and techniques

Methods of proving $\forall x\ p(x) \to q(x)$

Trivial proof	q is known
Vacuous proof	$\neg p$ is known
Direct proof	Assume p. Show q.
Proof by contraposition	Assume $\neg q$. Show $\neg p$.
Proof by contradiction	Assume the statement is false and derive a contradiction. Assume $\neg q \land p$. Show $r \land \neg r$
Proof by cases	p can be broken up in to cases ($p_1 \lor p_2 \lor \dots p_n$) Prove each case $p_i \to q$

Use <u>WLOG</u> (without loss of generality) before an assumption to narrow down the premise to some special case which can be easily applied to other similar cases

Steps to prove the biconditional: $p \leftrightarrow q$

1. Use any method to prove $p \to q$
2. Use any method to prove $q \to p$

Methods of proving existence: $\exists x\ P(x)$

Constructive	Find an explicit value *c* for which $P(c)$ is true.
Nonconstructive	Assume no c exists which P(c) is true and derive contradiction. Assume $\forall x \neg P(x)$. Show $r \land \neg r$

[there exists one and <u>only one</u> x such that P(x)]

Steps to prove unique existence: $\exists x\ P(x) \land (\forall y\ P(y) \to y=x)$

1. Prove existence. $\exists x\ P(x)$
2. Prove uniqueness. $\forall y\ P(y) \to y=x$