Discrete Structures Homework 2

Show your work to receive full credit.

Section 1.3

4. Use truth tables to verify the associative laws.
   a) $(p \lor q) \lor r \equiv p \lor (q \lor r)$
   b) $(p \land q) \land r \equiv p \land (q \land r)$

8. Use De Morgan's laws to find the negation of each of the following statements.
   a) Kwame will take a job in industry or go to graduate school.
   b) Yoshiko knows Java and calculus.
   c) James is young and strong.
   d) Rita will move to Oregon or Washington.

14. Determine whether $(\neg p \land (p \rightarrow q)) \rightarrow \neg q$ is a tautology

24. Show that $(p \rightarrow q) \lor (p \rightarrow r)$ and $p \rightarrow (q \lor r)$ are logically equivalent.

Section 1.4

6. Let $N(x)$ be the statement "x has visited North Dakota," where the domain consists of the students in your school. Express each of these quantifications in English.
   a) $\exists x N(x)$ b) $\forall x N(x)$ c) $\neg\exists x N(x)$
   d) $\exists x \neg N(x)$ e) $\neg\forall x N(x)$ f) $\forall x \neg N(x)$

10. Let $C(x)$ be the statement "x has a cat," let $D(x)$ be the statement "x has a dog," and let $F(x)$ be the statement "x has a ferret." Express each of these statements in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class.
   a) A student in your class has a cat, a dog, and a ferret.
   b) All students in your class have a cat, a dog, or a ferret.
   c) Some student in your class has a cat and a ferret, but not a dog.
   d) No student in your class has a cat, a dog, and a ferret.
   e) For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.

12. Let $Q(x)$ be the statement "$x + 1 > 2x$." If the domain consists of all integers, what are these truth values?
   a) $Q(0)$ b) $Q(-1)$ c) $Q(1)$
   d) $\exists x Q(x)$ e) $\forall x Q(x)$ f) $\exists x \neg Q(x)$
   g) $\forall x \neg Q(x)$

24. Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.

a) Everyone in your class has a cellular phone.
b) Somebody in your class has seen a foreign movie.
c) There is a person in your class who cannot swim.
d) All students in your class can solve quadratic equations.
e) Some student in your class does not want to be rich.

32. Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase "It is not the case that.")

a) All dogs have fleas.
b) There is a horse that can add.
c) Every koala can climb.
d) No monkey can speak French.
e) There exists a pig that can swim and catch fish.

44. Determine whether $\forall x(P(x) \leftrightarrow Q(x))$ and $\forall x P(x) \leftrightarrow \forall x Q(x)$ are logically equivalent. Justify your answer. (Hint: Find a counterexample by using propositional functions $P(x)$ and $Q(x)$ which are sometimes, but not always, true.)