An experiment is a procedure that yields one of a given set of possible outcomes.
The sample space (S) of the experiment is the set of possible outcomes.
An event (E) is a subset of the sample space.

assumes outcomes are equally likely

Laplace's Definition: If S is a finite sample space of equally likely outcomes, and E is an event (a subset of S), then the probability of E is

$$p(E) = \frac{|E|}{|S|}$$

- For every event E, $0 \leq p(E) \leq 1$.

doesn't assume outcomes are equally likely

General definition:

- Assigning probabilities: $0 \leq p(s) \leq 1$ for each $s \in S$, and $\sum_{s \in S} p(s) = 1$
  - $p(S)$ is the probability distribution
  - The uniform distribution assigns the probability 1/n to each $s \in S$
- The probability of an event is $p(E) = \sum_{s \in E} p(s)$

Complements: $p(\overline{E}) = 1 - p(E).$

Unions: $p(E_1 \cup E_2) = p(E_1) + p(E_2) - p(E_1 \cap E_2)$

- If $E_1, E_2, \ldots$ is a sequence of pairwise disjoint events, then

$$p\left(\bigcup_i E_i\right) = \sum_i p(E_i)$$

Conditional Probability: Let *E* and *F* be events with $p(F) > 0$. The conditional probability of *E* given *F* is

$$p(E|F) = \frac{p(E \cap F)}{p(F)}$$

Independence: The events *E* and *F* are independent if and only if $p(E \cap F) = p(E)p(F)$.

- $E_1, E_2, \ldots, E_n$ are pairwise independent if and only if $p(E_i \cap E_j) = p(E_i) \cdot p(E_j)$ for all pairs i and j with $i \leq j \leq n$.
- $E_1, E_2, \ldots, E_n$ are mutually independent if for all *m* such that $2 \leq m \leq n$, $p(E_1 \cap E_2 \cap \ldots \cap E_m) = p(E_1) \cdot p(E_2) \cdot \ldots \cdot p(E_m)$

- A Bernoulli trial is a random experiment with exactly two possible outcomes (success with probability *p*, and failure with probability *q*)
- The probability of exactly *k* successes in *n* independent Bernoulli trials is $C(n,k)p^k q^{n-k}$

- A random variable X is a function from the sample space to the set of real numbers.
- The expected value of a random variable X is equivalently:
  - $E(X) = \sum_{s \in S} p(s)X(s)$ (calculated per outcome)
  - $E(X) = \sum_{r \in X(S)} p(X = r)r.$ (calculated per random var value)
- When X represents the number of successes after n Bernoulli trials, $E(X) = np$
- $E(X_1 + X_2 + \ldots + X_n) = E(X_1) + E(X_2) + \ldots + E(X_n)$
- $E(aX + b) = aE(X) + b.$
- Random variables X and Y are independent if $p(X=r_1 \text{ and } Y=r_2) = p(X=r_1) \cdot p(Y=r_2)$
- If X and Y are independent variables on a sample space S, then $E(X \cdot Y) = E(X) \cdot E(Y).$