Recursive Definition

Proof technique for it

Recursively defined functions

- Basis Step: Specify the value of the function at 0.
- Recursive Step: Give a rule for finding f(n+1) from the function's value at smaller integers.

proving things about recursively defined functions (or recurrent relations describing sequences) →

Mathematical Induction Rule of Inference

$(P(b) \land \forall k \geq b\ (P(k) \rightarrow P(k + 1))) \rightarrow \forall n \geq b\ P(n)$,

Prove by mathematical induction that $\forall n \geq b\ P(n)$

- Basis Step: Prove P(b).
- Inductive Step: Prove $P(k) \rightarrow P(k+1)$ for $k \geq b$. Use a direct proof. Assume the inductive hypothesis - that P(k) for $k \geq b$. Use this to show P(k+1).

By mathematical induction, P(n) is true for all $n \geq b$.

Recursively defined sets

- Basis Step: Specify an initial collection of elements.
- Recursive Step: Give a rule for forming new elements in the set from those already known to be in the set

proving things about recursively defined sets or structures →

Prove by structural induction that P(n) for all sets or structures S

- Basis Step: Prove P(n) is true for all elements specified in the basis step of the recursive definition of S
- Recursive Step: Prove that if P(n) is true for each of the elements used to construct new elements in the recursive step of the definition of S, then P(n) is also true for the new elements created. Use a direct proof.

By structural induction, P(n) is true for all sets/ structures S