Terminology	Notation	Definition	Meaning
C is a subset of D	C⊆D	∀x(x∈C → x∈D)	Every element of C is also an element of D
C is a proper subset of D	C⊂D	∀x(x∈C → x∈D) ∧ ∃x(x∈D ∧ x∉C)	C⊆D but C≠D. Every element of C is also an element of D, but D has at least one element that C doesn't have
C is equal to D	C=D	∀x(x∈C ↔ x∈D)	C and D have exactly the same elements

N = natural numbers {0,1,2,…}
Z = integers {…,-1,0,1,…}
$\mathbf{Z^+}$ = positive integers {1,2,3,..}
R = real numbers (ex: 1.5, -π, 40)
$\mathbf{R^+}$ = positive real numbers (ex: π, 4.2)
Q = rational numbers
U = the universal set
∅ = the empty set

Three different ways to prove A=B
1. Prove that both A⊆B and B⊆A.
2. Use set builder notation and propositional logic.
3. Membership tables.

Set Identities & Operations:

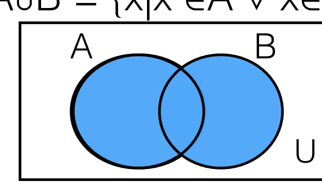
Law	Union	Intersection
Identity	A∪∅=A	A∩U=A
Domination	A∪U=U	A∩∅=∅
Idempotent	A∪A=A	A∩A=A
Double complement	$\overline{(\overline{A})}=A$	
Communtative	A∪B=B∪A	A∩B=B∩A
Associative	A∪(B∪C)=(A∪B)∪C	A∩(B∩C)=(A∩B)∩C
Distributive	A∪(B∩C)=(A∩B)∪(A∩C)	A∩(B∪C)=(A∪B)∩(A∪C)
De Morgan's	$\overline{A\cup B} = \overline{A} \cap \overline{B}$	$\overline{A\cap B} = \overline{A} \cup \overline{B}$
Absorption	A∪(A∩B)=A	A∩(A∪B)=A
Complement	$A\cup\overline{A} = U$	$A\cap\overline{A} =\varnothing$

Key Set Concepts:

- A set is an unordered collection of objects. It can be described via:
  - roster method: list all elements
  - set builder notation: S = {x | P(x)}
- x ∈ S means x is an element of S.
- x ∉ S means x is not an element of S.
- For every set S, ∅ ⊆ S and S ⊆ S.
- The cardinality of a finite set S, denoted |S|, is the number of distinct elements of S.
- The power set of S, denoted *P*(S), is the set of all subsets of S. If |S|=*n*, then $|P(S)|=2^n$.
- An n-tuple is an ordered collection of *n* objects, denoted as $(a_1, a_2, \ldots, a_n)$
  - two n-tuples are equal if and only if their corresponding elements are equal
- The Cartesian Product AxB is the set of all ordered pairs between elements of A and elements of B.
  - AxB = {(a,b) | a∈A ∧ b∧B}
- The Cartesian Product $A_1xA_2x\ldots xA_n$ is the set of all ordered n-tuples between elements of $A_1, A_2, \ldots A_n$.
  - $A_1xA_2x\ldots xA_n = \{(a_1,a_2,\ldots,a_n) \mid a_j\in A_j$ for $j=1,2,\ldots n)$
- ∀x ∈ S (P(x)) is shorthand for ∀x(x ∈ S → P(x))
- ∀x ∈ S (P(x)) is shorthand for ∀x(x ∈ S → P(x))
- The truth set of some predicate P(x) for a domain D is defined as {x∈D | P(x)}
- The Inclusion-Exclusion principle states that |A∪B|=|A|+|B|-|A∩

Union

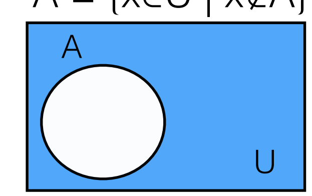
A∪B = {x|x ∈A ∨ x∈B}



$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \ldots \cup A_n$$

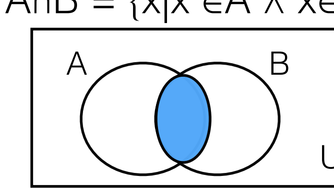
Complement

$\overline{A}$ = {x∈U | x∉A}



Intersection

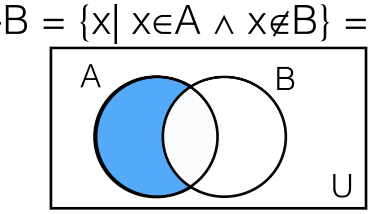
A∩B = {x|x ∈A ∧ x∈B}



$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \ldots \cap A_n$$

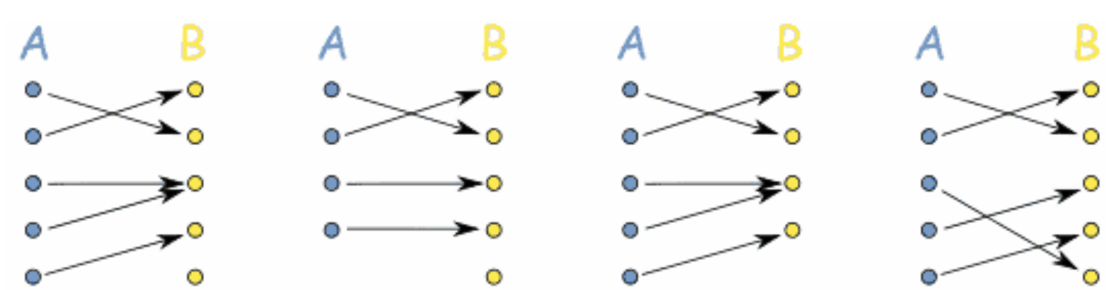
Difference

A-B = {x| x∈A ∧ x∉B} = $A\cap\overline{B}$



Key Function Concepts:

- A <u>function</u> *f* from a set A to B, denoted *f: A→B*, is an an assignment of each element of A to exactly one element of B
  - A is the <u>domain</u> of *f*
  - B is the <u>codomain</u> of *f*
  - f(A) is the <u>range</u> of *f*
  - If *f(a)=b*, then *b* is the <u>image</u> of *a* under *f*, and *a* is the <u>preimage</u> of *b*.
- Two functions are <u>equal</u> when they have the same domain, same codomain, and map each element of the domain to the same element of the codomain.
- A function can be represented via:
  - explicit statement of assignments
  - a formula
  - computer program
- Let *f: B→C* and *g: A→B*. The <u>composition</u> of *f* with *g* is f∘g: A→*C*, where (f∘g)(a)=f(g(a)).
- The <u>floor</u> function, denoted $f(x)=\lfloor x \rfloor$, is the largest integer ≤ x.
- The <u>ceiling</u> function, denoted $f(x)=\lceil x \rceil$, is the smallest integer ≥ x.
- Given a bijective function *f:C→D*, the <u>inverse</u> f^{-1}:*D→C* is defined as $f^{-1}(y)=x$ if and only if f(x)=y. No inverse exists unless *f* is bijective.
- A function *f:C→D* is <u>injective</u> (one-to-one) if and only if ∀xy∊C f(x)=f(y)→x=y
- A function *f:C→D* is <u>surjective</u> (onto) if and only if ∀y∊D ∃x∊C f(x)=y. In this instance, f(C)=D.
- A function *f:C→D* is <u>bijective</u> (one-to-one correspondence) if and only if *f* is both injective and surjective.



general function | injective (not surjective) | surjective (not injective) | bijective (injective and surjective)

Key Sequence Concepts:

- A <u>sequence</u> $\{a_n\}$ is a function from the subset of integers to the set S. It provides an ordered list of elements.
- a_n is used to represent f(n), and is called the *n*th <u>term</u> of the sequence.
- A <u>geometric progression</u> is a sequence of the form a, ar, ar^2, ar^3, …, ar^n
- An <u>arithmetic progression</u> is a sequence of the form a, a+d, a+2d,…, a+nd
- A <u>string</u> is a finite sequence of characters from a finite set (an alphabet)
  - the <u>empty string</u> is represented by λ
  - the <u>length</u> of a string is the number of characters in it
- A <u>recurrence relation</u> for a sequence $\{a_n\}$ is an equation that expresses a_n in terms of one or more of previous terms of the sequence
  - It requires <u>initial conditions</u> which specify the terms that precede the first term where the recurrence relation takes effect
  - A sequence is a <u>solution</u> of a recurrence relation if its terms satisfy it
- The <u>Fibonacci sequence</u> f_0, f_1, f_2, … is defined by
  - initial conditions: $f_0=0$ and $f_1=1$
  - recurrence relation: $f_n=f_{n-1} + f_{n-2}$
- Solve the recurrence relation which generates a sequence by finding a <u>closed formula</u> for the nth term of the sequence (doesn't rely on previous terms), using an iterative solution of either forward substitution or backwards substitution
- Sum of terms $a_m, a_{m+1}, \ldots, a_n$ from the sequence $\{a_n\}$ is denoted by $\sum_{j=m}^{n} a_j$

TABLE 2 Some Useful Summation Formulae.

Sum	*Closed Form*
$\sum_{k=0}^{n} ar^k \ (r \neq 0)$	$\frac{ar^{n+1}-a}{r-1}, r \neq 1$
$\sum_{k=1}^{n} k$	$\frac{n(n+1)}{2}$
$\sum_{k=1}^{n} k^2$	$\frac{n(n+1)(2n+1)}{6}$
$\sum_{k=1}^{n} k^3$	$\frac{n^2(n+1)^2}{4}$
$\sum_{k=0}^{\infty} x^k, \lvert x \rvert < 1$	$\frac{1}{1-x}$
$\sum_{k=1}^{\infty} kx^{k-1}, \lvert x \rvert < 1$	$\frac{1}{(1-x)^2}$