Applications of Propositional Logic

Section 1.2

Applications of Propositional Logic: Summary

- Translating English to Propositional Logic
- System Specifications
- Logic Puzzles
- Logic Circuits

Translating English Sentences

- Steps to convert an English sentence to a statement in propositional logic
  - Identify atomic propositions and represent using propositional variables.
  - Determine appropriate logical connectives
- "If I go to Harry's or to the country, I will not go shopping."
  - *p*: I go to Harry's
  - q: I go to the country.
  - *r*: I will go shopping.

If *p* or *q* then not *r*.

$$(p \vee q) \rightarrow \neg r$$

Example: Translate into propositional logic

"You can access the Internet from campus only if you are a computer science major or you are not a freshman."

One Solution:

a: "You can access the internet from campus"

c: "You are a computer science major"

f: "You are a freshman."

$$a \rightarrow (c \lor \neg f)$$

System Specifications

- System and Software engineers take requirements in English and express them in a precise specification language based on logic.

Example: Express in propositional logic:

"The automated reply cannot be sent when the file system is full"

One solution: Let *p* denote "The automated reply can be sent" and *q* denote "The file system is full."

$$q \rightarrow \neg p$$

Consistent System Specifications

Definition: A list of propositions is *consistent* if it is possible to assign truth values (T/F) to the proposition variables so that each compound proposition in the list is true.

Exercise: Are these specifications consistent?

- "The diagnostic message is stored in the buffer or it is retransmitted." $p \vee q$
- "The diagnostic message is not stored in the buffer." $\neg p$
- "If the diagnostic message is stored in the buffer, then it is retransmitted." $p \rightarrow q$

Solution: *p*: "The diagnostic message is stored in the buffer."

q: "The diagnostic message is retransmitted."

When *p* is false and *q* is true all three statements are true. So the specification is consistent.

Consistent System Specifications

Exercise: What if the specification "The diagnostic message is not retransmitted" is added? Is it still consistent?

- "The diagnostic message is stored in the buffer or it is retransmitted." $p \vee q$
- "The diagnostic message is not stored in the buffer." $\neg p$
- "If the diagnostic message is stored in the buffer, then it is retransmitted." $p \rightarrow q$
- "The diagnostic message is not retransmitted." $\neg q$

Solution: There is no satisfying assignment. The specification is not consistent.

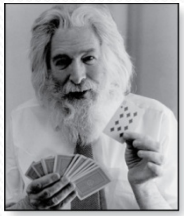
Consistent System Specifications

- “The diagnostic message is stored in the buffer or it is retransmitted.”
- “The diagnostic message is not stored in the buffer.”
- “If the diagnostic message is stored in the buffer, then it is retransmitted.”

- What if “The diagnostic message is not retransmitted” is added.

p	q	$p \vee q$	$\neg p$	$p \rightarrow q$	$\neg q$
F	F	F	T	T	T
F	T	T	T	T	F
T	F	T	F	F	T
T	T	T	F	T	F

Logic Puzzles



Raymond Smullyan (Born 1919)

- An island has two kinds of inhabitants, *knights*, who always tell the truth, and *knaves*, who always lie.
- You go to the island and meet A and B.
  - A says “B is a knight.”
  - B says “The two of us are of opposite types.”

Example: What are the types of A and B?

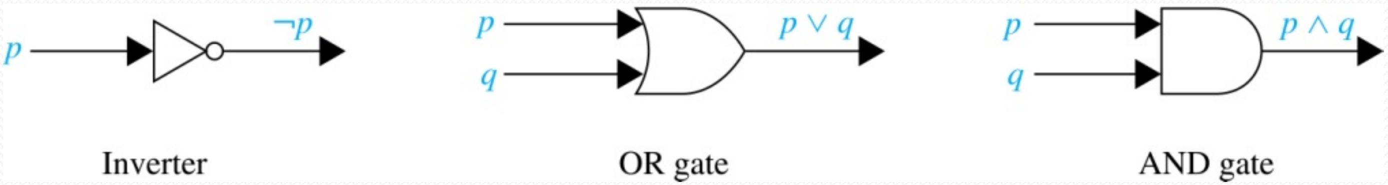
Solution: Let *p: “A is a knight”* and *q: “B is a knight.*

So, then $\neg p$: “*A is a knave*” and $\neg q$: “*B is a knave.*”

- If A is a knight, then *p* is true. Since knights tell the truth, *q* must also be true. Then $(p \wedge \neg q) \vee (\neg p \wedge q)$ would have to be true, but it is not. So, A is not a knight and therefore $\neg p$ must be true.
- If A is a knave, then B must not be a knight since knaves always lie. So, then both $\neg p$ and $\neg q$ hold since both are knaves.

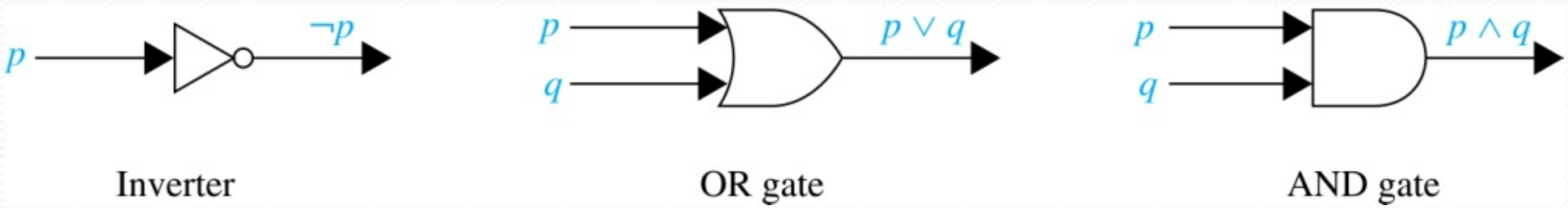
Logic Circuits (Studied in depth in Chapter 12)

- Electronic circuits; each input/output signal can be viewed as a 0 or 1.
  - 0 represents **False/Off**
  - 1 represents **True/On**
- Complicated circuits are constructed from three basic circuits called gates.



  - The inverter (**NOT gate**)takes an input bit and produces the negation of that bit.
  - The **OR gate** takes two input bits and produces the value equivalent to the disjunction of the two bits.
  - The **AND gate** takes two input bits and produces the value equivalent to the conjunction of the two bits.

Logic Circuits (Studied in depth in Chapter 12)



- More complicated digital circuits can be constructed by combining these basic circuits to produce the desired output given the input signals by building a circuit for each piece of the output expression and then combining them.
- For example, this circuit results in $(p \wedge \neg q) \vee \neg r$