

For each of the following recurrence equations which describe the running time  $T(n)$  of a recursive algorithm, use the master method to express the asymptotic complexity (assuming that  $T(n) = c$  for  $n < d$ , for constants  $c > 0$  and  $d \geq 1$ ).

**Example 1.**  $T(n) = 9T(n/3) + n^2$

Then,  $f(n) = n^2$  and  $n^{\log_b(a)} = n^{\log_3(9)} = n^2$ .

By case 2 (with  $k = 0$ ) of the master theorem,  $T(n) = \Theta(n^2 \log n)$ .

**Example 2.**  $T(n) = 4T(n/2) + n$

Then,  $f(n) = n$  and  $n^{\log_b(a)} = n^{\log_2(4)} = n^2$ .

By case 1 of the master theorem,  $T(n) = \Theta(n^2)$ .

**Example 3.**  $T(n) = 8T(n/8) + n^2 \log n$

Then,  $f(n) = n^2 \log n$  and  $n^{\log_b(a)} = n^{\log_8(8)} = n$ .

By case 3 of the master theorem,  $T(n) = \Theta(n^2 \log n)$ .

**Example 4.**  $T(n) = 4T(n/4) + n$

Then,  $f(n) = n$  and  $n^{\log_b(a)} = n^{\log_4(4)} = n$ .

By case 2 of the master theorem (with  $k = 0$ ),  $T(n) = \Theta(n \log n)$ .

**Example 5.**  $T(n) = 4T(n/2) + n^2 \log^3 n$

Then,  $f(n) = n^2 \log^3 n$  and  $n^{\log_b(a)} = n^{\log_2(4)} = n^2$ .

By case 2 (with  $k = 3$ ) of the master theorem,  $T(n) = \Theta(n^2 \log^4 n)$ .

**Example 6.**  $T(n) = 4T(n/2) + \log n$

Then,  $f(n) = \log n$  and  $n^{\log_b(a)} = n^{\log_2(4)} = n^2$ .

By case 1 of the master theorem,  $T(n) = \Theta(n^2)$ .

**Example 7.**  $T(n) = 4T(n/4) + n^2$

Then,  $f(n) = n^2$  and  $n^{\log_b(a)} = n^{\log_4(4)} = n$ .

By case 3 of the master theorem,  $T(n) = \Theta(n^2)$ .

**Example 8.**  $T(n) = 7T(n/3) + \log n$

Then,  $f(n) = \log n$  and  $n^{\log_b(a)} = n^{\log_3(7)} = n^{1.7712437}$ .

By case 1 of the master theorem,  $T(n) = \Theta(n^{\log_3(7)})$ .