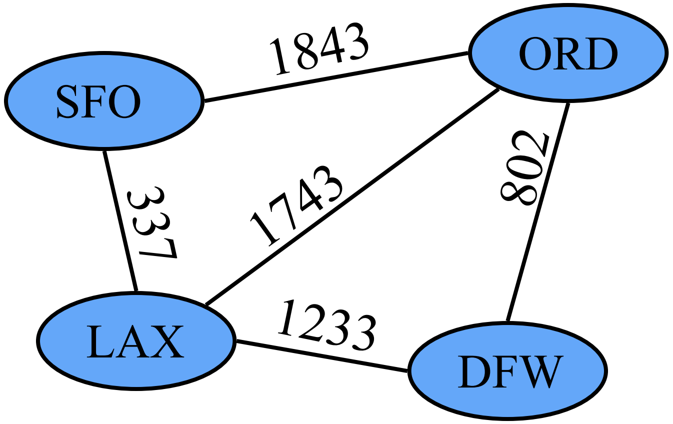
# Graphs



# Outline / Reading

Graphs (6.1)
- Definition
- Applications
- Terminology
- Properties
- ADT

Data structures for graphs (6.2)
- Edge list structure
- Adjacency list structure
- Adjacency matrix structure

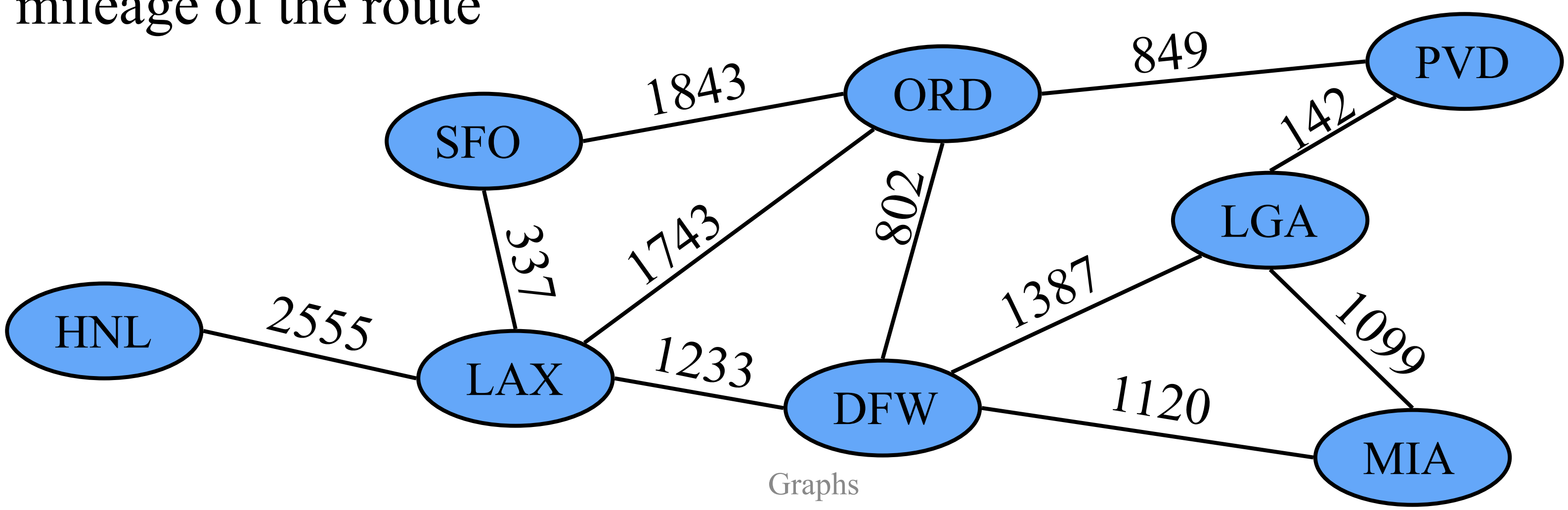
# Graph

A graph is a pair (***V, E***), where

- ***V*** is a set of nodes, called vertices
- ***E*** is a collection of pairs of vertices, called edges
- Vertices and edges are positions and store elements

Example:

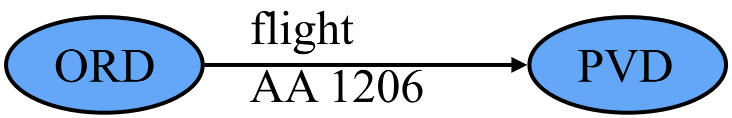
- A vertex represents an airport and stores the three-letter airport code
- An edge represents a flight route between two airports and stores the mileage of the route



# Edge Types

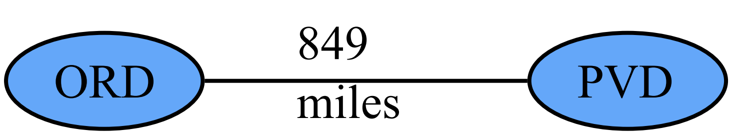
Directed edge
- ordered pair of vertices (***u***,***v***)
- first vertex ***u*** is the origin
- second vertex ***v*** is the destination
- e.g., a flight

Directed graph
- all the edges are directed
- e.g., flight network



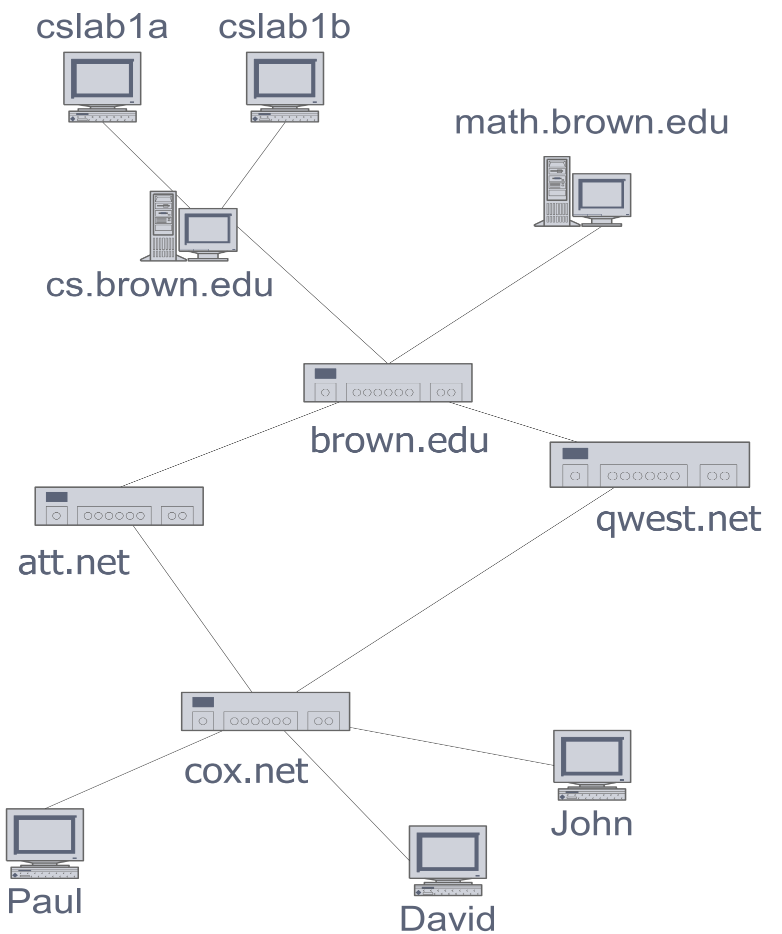

Undirected edge
- unordered pair of vertices (***u***,***v***)
- e.g., a flight route

Undirected graph
- all the edges are undirected
- e.g., route network



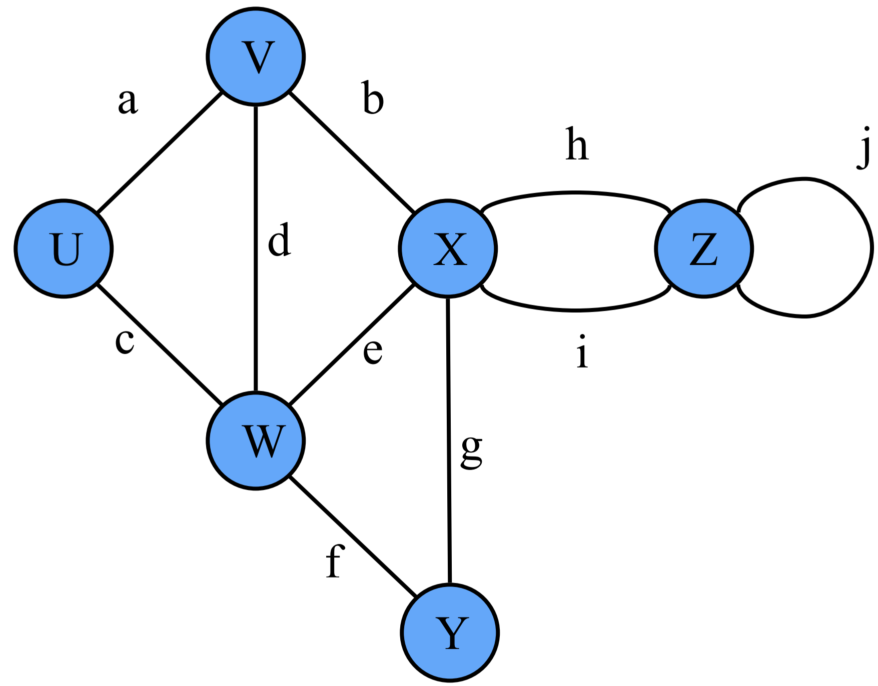
# Applications

- Electronic circuits
  - Printed circuit board
  - Integrated circuit
- Transportation networks
  - Highway network
  - Flight network
- Computer networks
  - Local area network
  - Internet
  - Web
- Databases
  - Entity-relationship diagram



# Terminology

- **End** vertices (or endpoints) of an edge
  - $U$ and $V$ are the endpoints of $a$
- Edges **incident** on a vertex
  - $a$, $d$, and $b$ are incident on $V$
- **Adjacent** vertices
  - $U$ and $V$ are adjacent
- **Degree** of a vertex
  - $X$ has degree 5
- **Parallel** edges
  - $h$ and $i$ are parallel edges
- **Self-loop**
  - $j$ is a self-loop



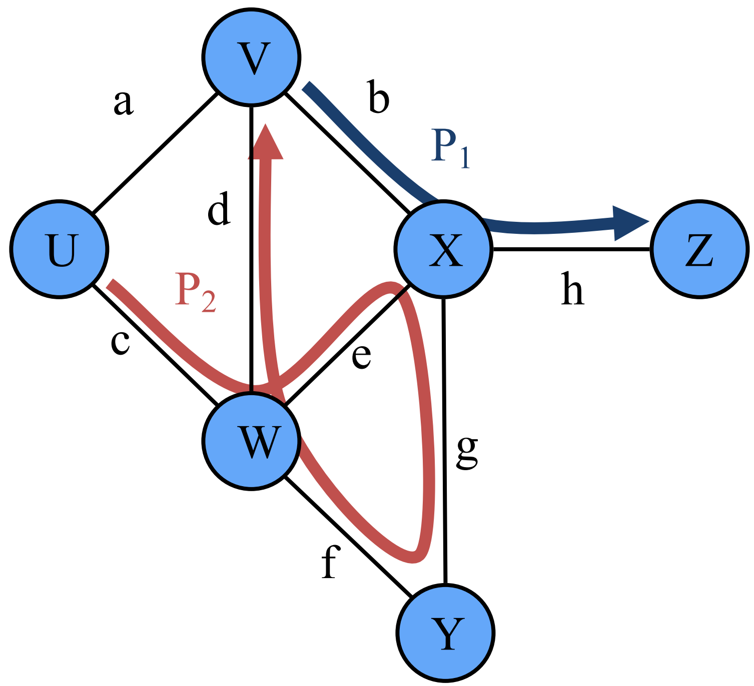
# Terminology (cont.)

**Path**

- sequence of alternating vertices and edges
- begins with a vertex
- ends with a vertex
- each edge is preceded and followed by its endpoints

**Simple path**

- path such that all its vertices and edges are distinct




Examples

- $P_1$=(V,b,X,h,Z) is a simple path
- $P_2$=(U,c,W,e,X,g,Y,f,W,d,V) is a path that is not simple

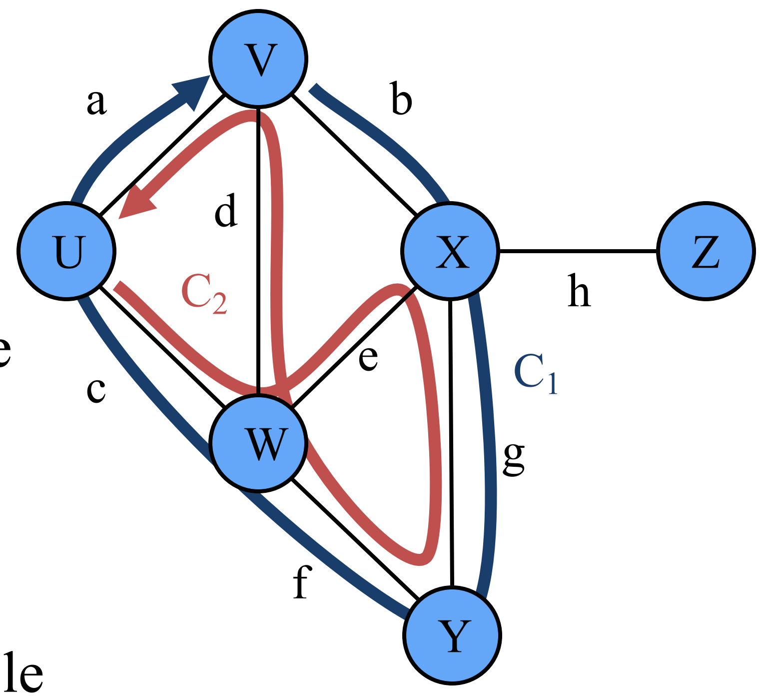
# Terminology (cont.)

**Cycle**

- circular sequence of alternating vertices and edges
- each edge is preceded and followed by its endpoints

**Simple cycle**

- cycle such that all its vertices and edges are distinct




Examples

- $C_1$=(V,b,X,g,Y,f,W,c,U,a,↵) is a simple cycle
- $C_2$=(U,c,W,e,X,g,Y,f,W,d,V,a,↵) is a cycle that is not simple

# Properties

Property 1. In an undirected graph

$$\Sigma_v \deg(v) = 2m$$

Proof: each edge is counted twice

Property 2. In an undirected graph with no self-loops and no multiple edges

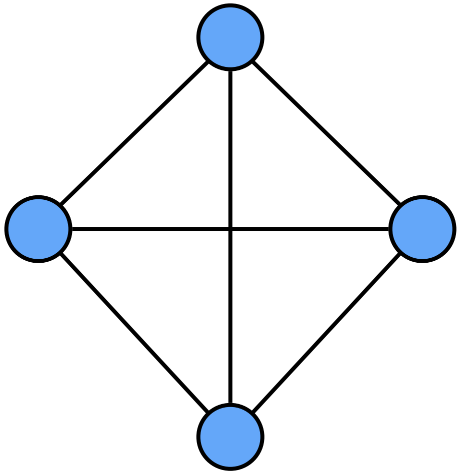
$$m \le n\,(n-1)/2$$

Proof: each vertex has degree at most $(n-1)$

What is the bound for a directed graph?

Notation

| | |
|---|---|
| $n$ | number of vertices |
| $m$ | number of edges |
| $\deg(v)$ | degree of vertex $v$ |



Ex: $n = 4$; $m = 6$; $\deg(v) = 3$

# Main Methods of the Graph ADT

Vertices and edges
- are positions
- store elements

Accessor methods
- aVertex()
- incidentEdges(v)
- endVertices(e)
- isDirected(e)
- origin(e)
- destination(e)
- opposite(v, e)
- areAdjacent(v, w)

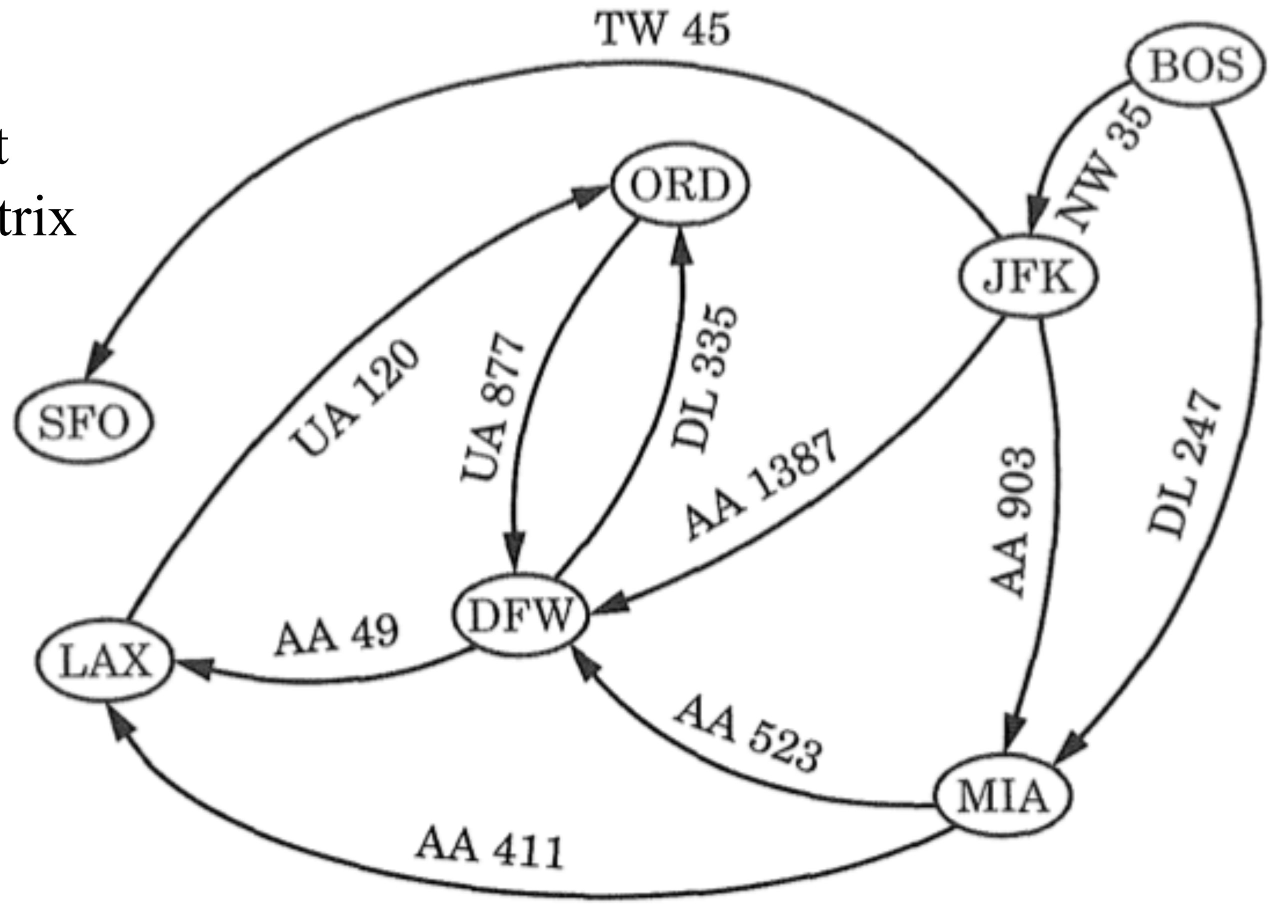
Update methods
- insertVertex(o)
- insertEdge(v, w, o)
- insertDirectedEdge(v, w, o)
- removeVertex(v)
- removeEdge(e)

Generic methods
- numVertices()
- numEdges()
- vertices()
- edges()

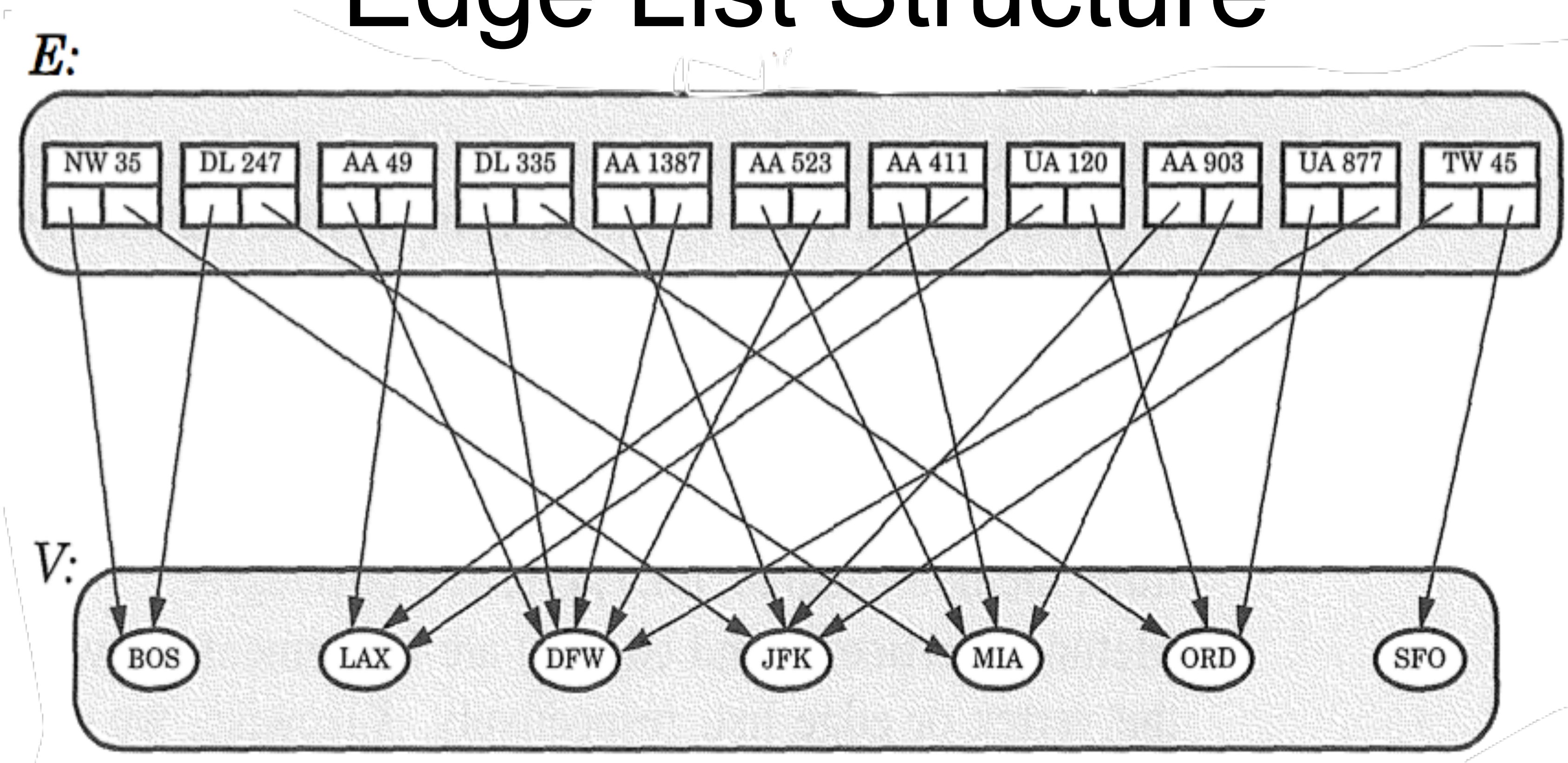
# Data Structures

Structures to represent a graph:

1. Edge List
2. Adjacency List
3. Adjacency Matrix

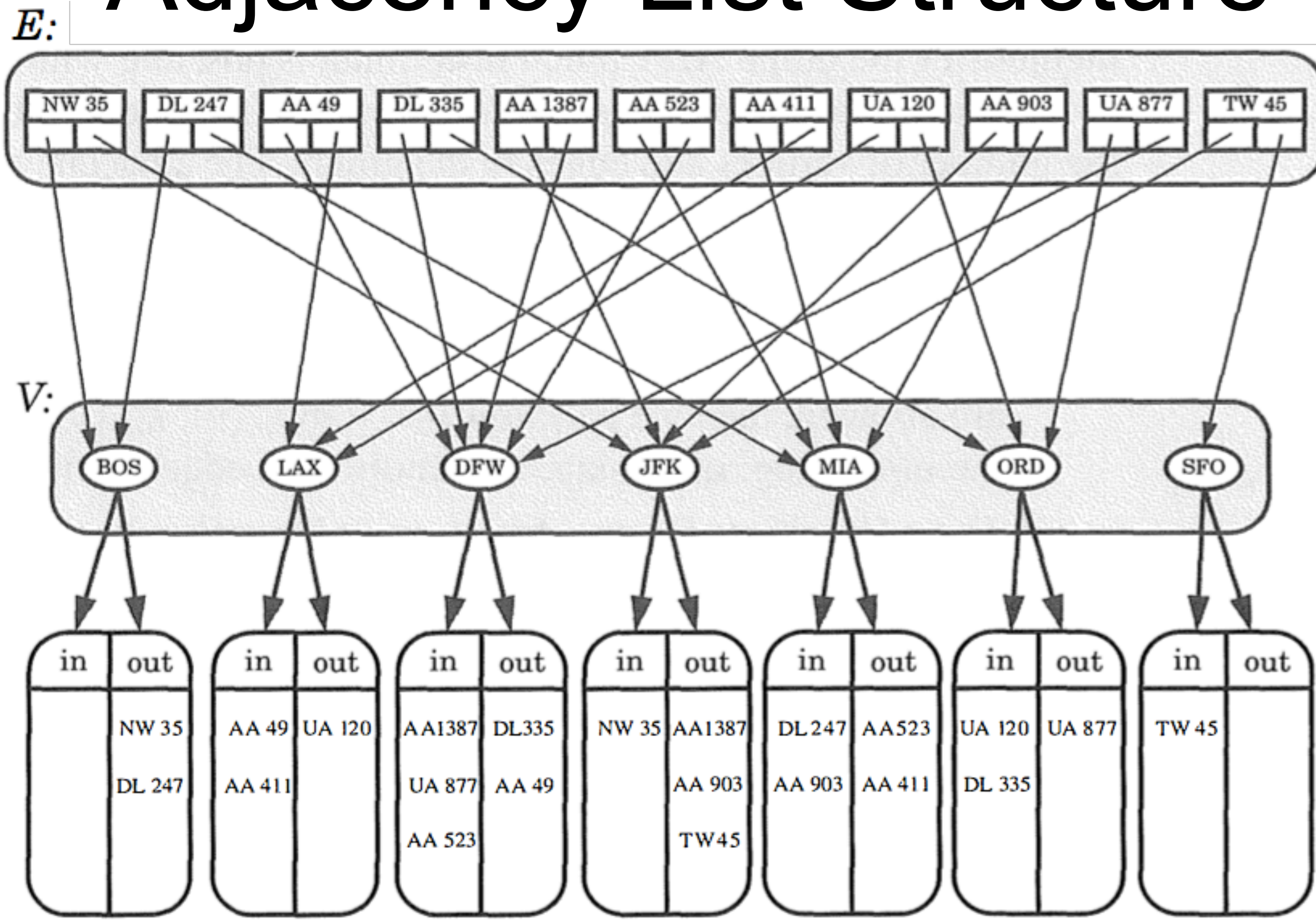


# Edge List Structure




A container of edge objects, where each edge object references the origin and destination vertex object

# Adjacency List Structure




An edge list structure, where additionally each vertex object *v* references an incidence container which stores references to the edges incident on *v*.

# Adjacency Matrix Structure

| | | 0 BOS | 1 DFW | 2 JFK | 3 LAX | 4 MIA | 5 ORD | 6 SFO |
|---|---|---|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| BOS | 0 | Ø | Ø | NW 35 | Ø | DL 247 | Ø | Ø |
| DFW | 1 | Ø | Ø | Ø | AA 49 | Ø | DL 335 | Ø |
| JFK | 2 | Ø | AA 1387 | Ø | Ø | AA 903 | Ø | TW 45 |
| LAX | 3 | Ø | Ø | Ø | Ø | Ø | UA 120 | Ø |
| MIA | 4 | Ø | AA 523 | Ø | AA 411 | Ø | Ø | Ø |
| ORD | 5 | Ø | UA 877 | Ø | Ø | Ø | Ø | Ø |
| SFO | 6 | Ø | Ø | Ø | Ø | Ø | Ø | Ø |

A 2D array of all vertex pairs, where cell A[$u$,$v$] stores edge $e$ incident on vertices $u,v$ if such an edge exists.

# Asymptotic Performance

| ◆ $n$ vertices, $m$ edges<br>◆ no parallel edges<br>◆ no self-loops<br>◆ Bounds are "big-Oh" | Edge List | Adjacency List | Adjacency Matrix |
|---|---|---|---|
| Space | $n + m$ | $n + m$ | $n^2$ |
| incidentEdges($v$) | $m$ | $\deg(v)$ | $n$ |
| areAdjacent ($v$, $w$) | $m$ | $\min(\deg(v), \deg(w))$ | 1 |
| insertVertex($o$) | 1 | 1 | $n^2$ |
| insertEdge($v$, $w$, $o$) | 1 | 1 | 1 |
| removeVertex($v$) | $m$ | $\deg(v)$ | $n^2$ |
| removeEdge($e$) | 1 | 1 | 1 |