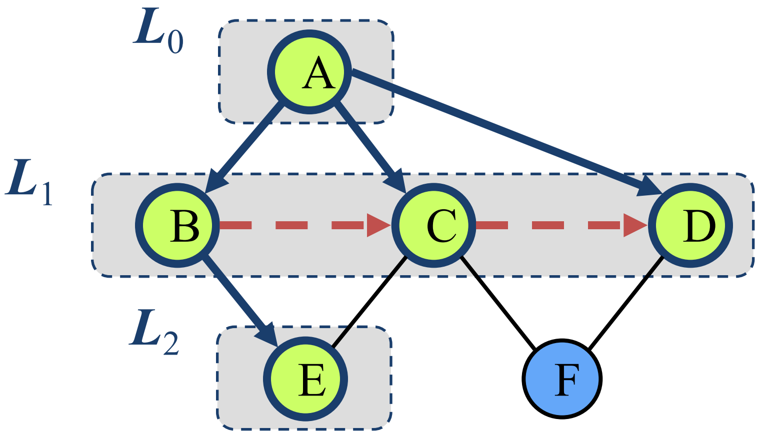
Breadth-First Search



Outline and Reading

Breadth-first search (6.3.3)

- Algorithm
- Example
- Properties
- Analysis
- Applications

DFS vs. BFS (6.3.3)

- Comparison of applications
- Comparison of edge labels

Breadth-First Search

- Breadth-first search (BFS) is a general technique for traversing a graph. A BFS traversal of a graph G
  - visits all the vertices and edges of G
  - determines whether G is connected
  - computes the connected components of G
  - computes a spanning forest of G

- BFS on a graph with n vertices and m edges takes $O(n + m)$ time

- BFS can be further extended to solve other graph problems
  - find and report a path with the minimum number of edges between two given vertices
  - find a simple cycle, if there is one

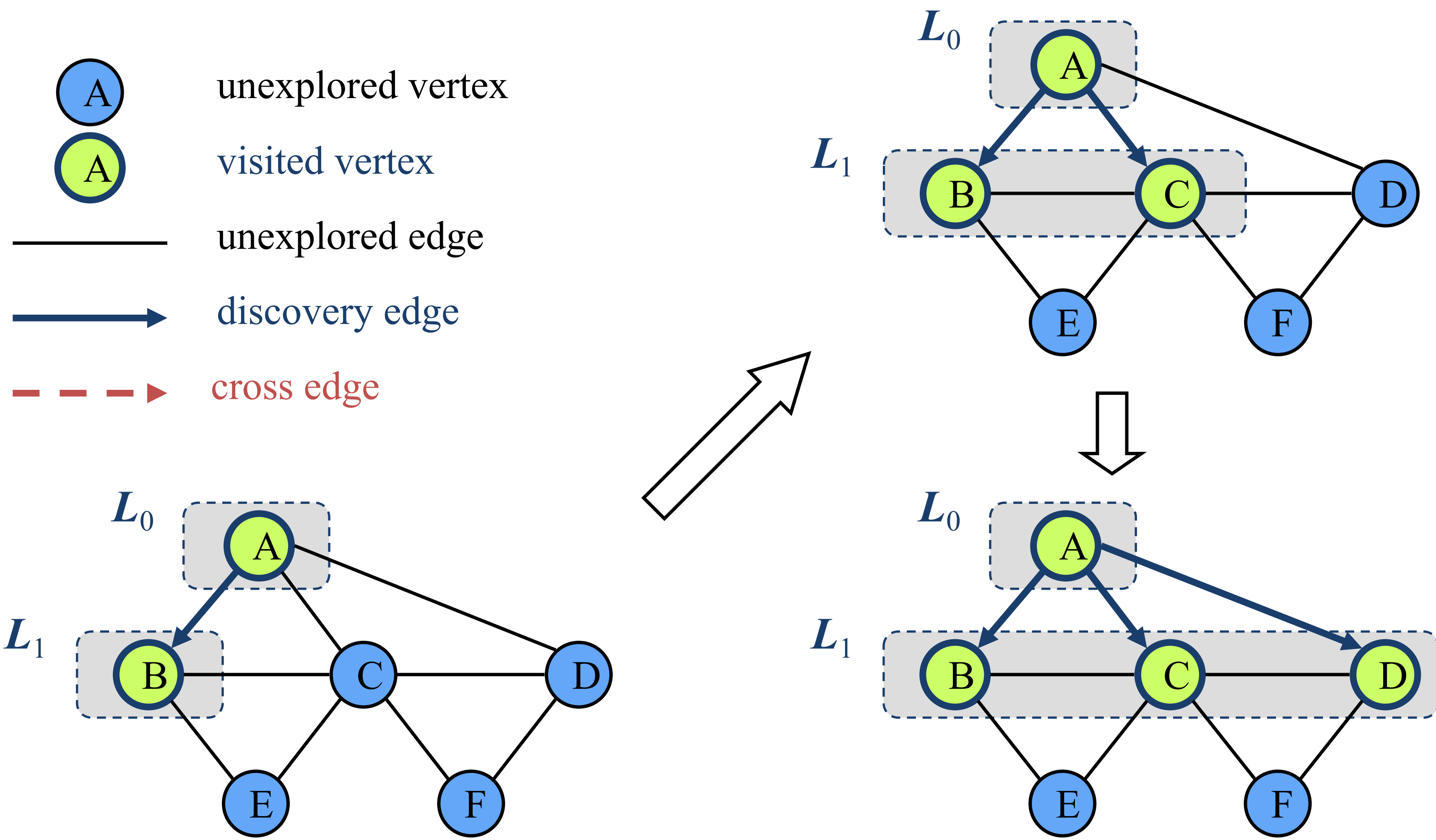
BFS Algorithm

The algorithm uses a mechanism for setting and getting "labels" of vertices and edges.

Algorithm *BFS*(*G*)
  Input graph *G*
  Output labeling of the edges and partition of the vertices of *G*
  for all $u \in G.vertices()$
    setLabel(*u, UNEXPLORED*)
  for all $e \in G.edges()$
    setLabel(*e, UNEXPLORED*)
  for all $v \in G.vertices()$
    if *getLabel*(*v*) = *UNEXPLORED*
      BFS(*G, v*)

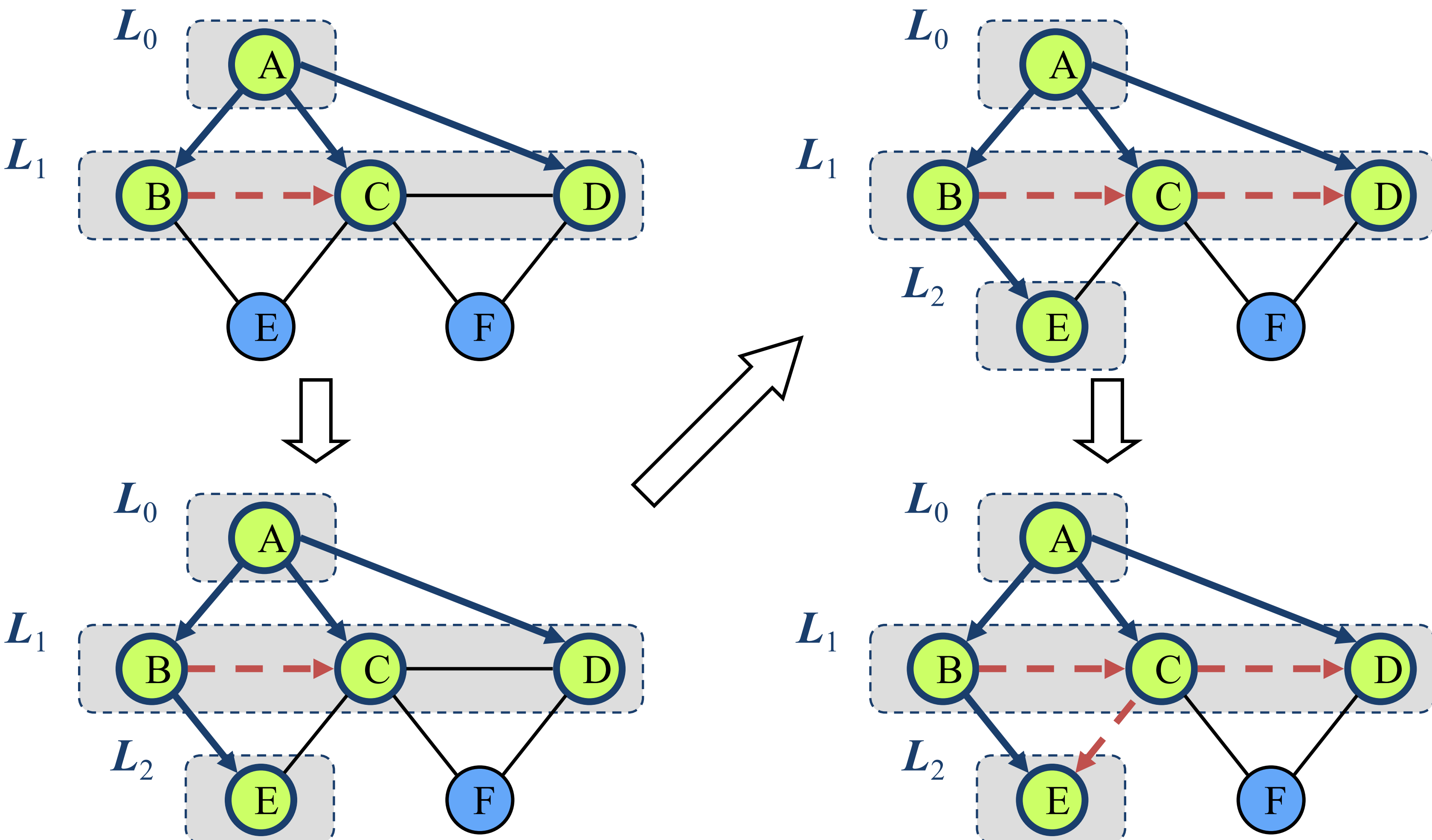
Algorithm *BFS*(*G, s*)
  $L_0 \leftarrow$ new empty sequence
  L_0.*insertLast*(*s*)
  setLabel(*s, VISITED*)
  $i \leftarrow 0$
  while $\neg L_i$.*isEmpty*()
    $L_{i+1} \leftarrow$ new empty sequence
    for all $v \in L_i.elements()$
      for all $e \in G.incidentEdges(v)$
        if *getLabel*(*e*) = *UNEXPLORED*
          $w \leftarrow opposite(v,e)$
          if *getLabel*(*w*) = *UNEXPLORED*
            setLabel(*e, DISCOVERY*)
            setLabel(*w, VISITED*)
            L_{i+1}.*insertLast*(*w*)
          else
            setLabel(*e, CROSS*)
    $i \leftarrow i + 1$

Example

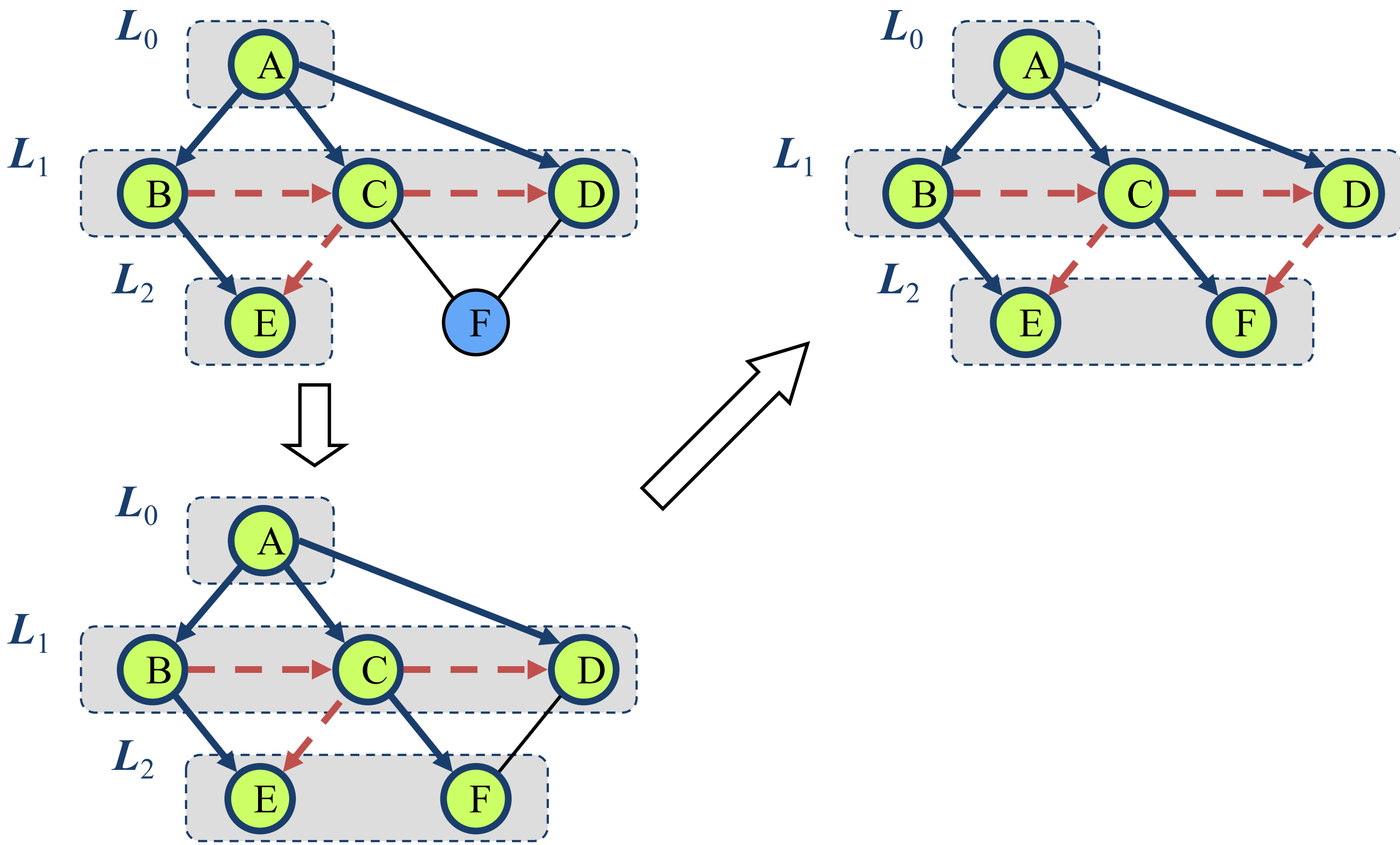


A
unexplored vertex
A
visited vertex
unexplored edge
discovery edge
cross edge
L_0
L_1
A
B
C
D
E
F
L_0
L_1
A
B
C
D
E
F
L_0
L_1
A
B
C
D
E
F

Example (cont.)



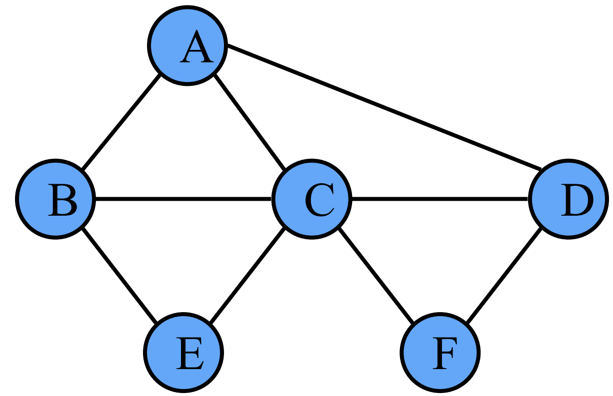
Example (cont.)



Properties

Notation

G_s: connected component of s

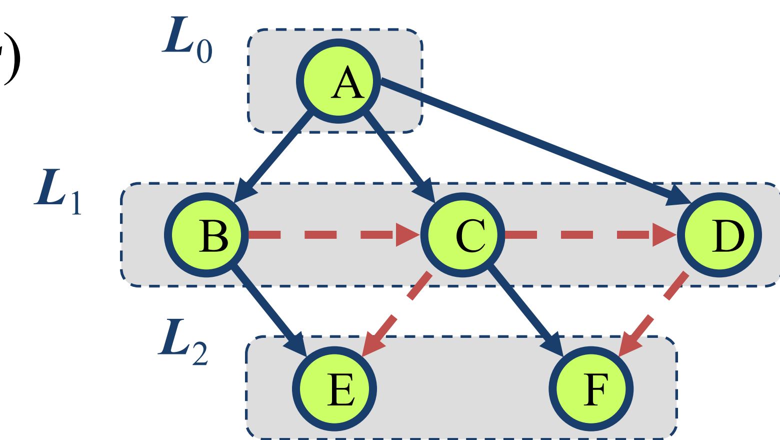


Property 1

$BFS(G, s)$ visits all the vertices and edges of G_s

Property 2

The discovery edges labeled by $BFS(G, s)$ form a spanning tree T_s of G_s



Property 3

For each vertex v in L_i

- The path of T_s from s to v has i edges
- Every path from s to v in G_s has at least i edges

Analysis

- Setting/getting a vertex/edge label takes $O(1)$ time
- Each vertex is labeled twice
  - once as UNEXPLORED
  - once as VISITED
- Each edge is labeled twice
  - once as UNEXPLORED
  - once as DISCOVERY or CROSS
- Each vertex is inserted once into a sequence L_i
- Method incidentEdges is called once for each vertex
- BFS runs in $O(n + m)$ time provided the graph is represented by the adjacency list structure
  - Recall that $\Sigma_v \deg(v) = 2m$

Applications

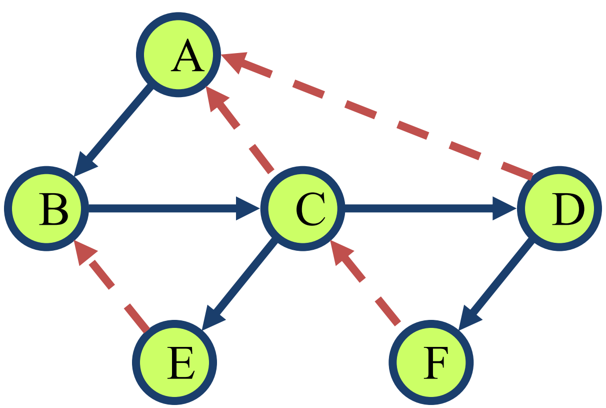
Using the template method pattern, we can specialize the BFS traversal of a graph $\boldsymbol{G}$ to solve the following problems in $\boldsymbol{O}(\boldsymbol{n} + \boldsymbol{m})$ time:

- Compute the connected components of $\boldsymbol{G}$
- Compute a spanning forest of $\boldsymbol{G}$
- Find a simple cycle in $\boldsymbol{G}$, or report that $\boldsymbol{G}$ is a forest
- Given two vertices of $\boldsymbol{G}$, find a path in $\boldsymbol{G}$ between them with the minimum number of edges, or report that no such path exists

DFS vs. BFS

Back edge (v,w)

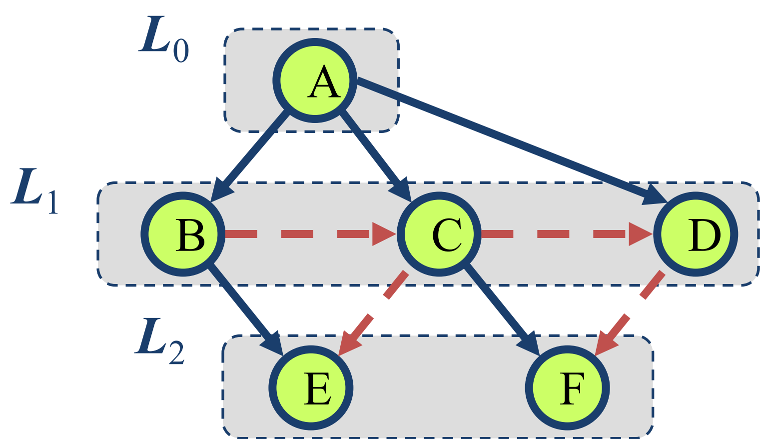
- w is an ancestor of v in the tree of discovery edges



DFS

Cross edge (v,w)

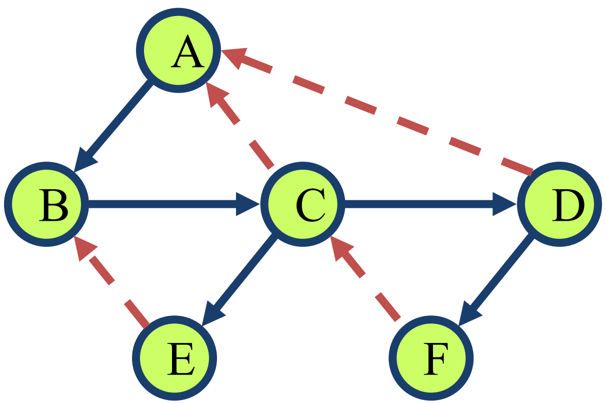
- w is in the same level as v or in the next level in the tree of discovery edges



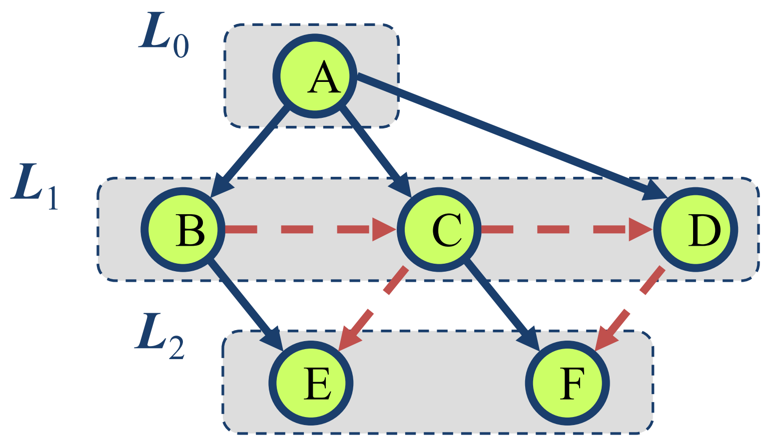
BFS

DFS vs. BFS

Applications	DFS	BFS
Spanning forest, connected components, paths, cycles	√	√
Shortest paths		√
Biconnected components	√	



DFS



BFS