

Sets

Set ADT

- A collection of unordered **distinct** objects
 - there is no inherent ordering of elements in a set, but keeping the elements sorted can lead to more efficient set operations
- Main operations
 - `union(B)`: executes $A \leftarrow A \cup B$
 - `intersect(B)`: executes $A \leftarrow A \cap B$
 - `subtract(B)`: executes $A \leftarrow A - B$
 - implemented using a generic version of the merge algorithm
- Running time of an operation should be at most $O(n_A + n_B)$

Storing a Set in a List

- We can implement a set with a list
- Elements are sorted according to some canonical ordering
- Space used is $O(n)$



Generic Merging

- Generalized merge of two sorted lists A and B
- Auxiliary methods `aIsLess`, `bIsLess`, `bothAreEqual` decide whether to add the element to list S based on what main operation is performed

Algorithm *genericMerge*(A, B)

$S \leftarrow$ empty sequence

while $\neg A.isEmpty() \wedge \neg B.isEmpty()$

$a \leftarrow A.first().element(); b \leftarrow B.first().element()$

if $a < b$

$aIsLess(a, S); A.remove(A.first())$

else if $b < a$

$bIsLess(b, S); B.remove(B.first())$

else { $b = a$ }

$bothAreEqual(a, b, S)$

$A.remove(A.first()); B.remove(B.first())$

while $\neg A.isEmpty()$

$aIsLess(a, S); A.remove(A.first())$

while $\neg B.isEmpty()$

$bIsLess(b, S); B.remove(B.first())$

return S

Example: Union

- if $a < b$, copy a to output sequence and go to next element of A
- if $a = b$, copy a to output sequence and go to next element of A and B
- if $a > b$, copy b to output sequence and go to next element of B



$$S = A \cup B$$

Example: Union

- if $a < b$, copy a to output sequence and go to next element of A
- if $a = b$, copy a to output sequence and go to next element of A and B
- if $a > b$, copy b to output sequence and go to next element of B

A

2	5	6	7	9
---	---	---	---	---

B

2	7	8	10
---	---	---	----

$S = A \cup B$

2

Example: Union

- if $a < b$, copy a to output sequence and go to next element of A
- if $a = b$, copy a to output sequence and go to next element of A and B
- if $a > b$, copy b to output sequence and go to next element of B

A

2	5	6	7	9
---	---	---	---	---

B

2	7	8	10
---	---	---	----

$S = A \cup B$

2	5
---	---

Example: Union

- if $a < b$, copy a to output sequence and go to next element of A
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- if $a > b$, copy b to output sequence and go to next element of B

A

2	5	6	7	9
---	---	---	---	---

B

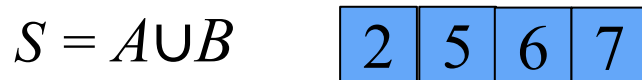
2	7	8	10
---	---	---	----

$S = A \cup B$

2	5	6
---	---	---

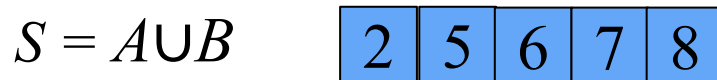
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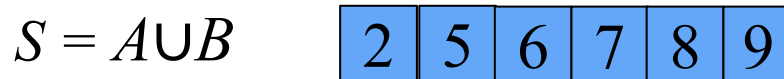
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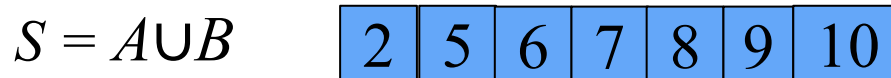
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Using Generic Merge for Set Operations

- Any of the set operations can be implemented using a generic merge
- For example:
 - intersection: only copy elements that are duplicated in both lists
 - subtraction: only copy elements from A that are not equal to those in B
- All methods run in linear time.