Dictionaries

Dictionary ADT

- Models a searchable collection of key-element items called entries
- Main operations: find, insert, remove
  - findElement(k), insertItem(k, o), removeElement(k)
  - size(), isEmpty()
  - keys(), elements()
- Applications:
  - address book
  - word-definition pairs
  - mapping host names to internet addresses (e.g., www.cs16.net to 128.148.34.101)

Log File

- A log file is a dictionary implemented by means of storing items in an unsorted sequence
  - insertItem takes $O(1)$ time since we can insert the new item at the beginning or at the end of the sequence
  - findElement and removeElement take $O(n)$ time since in the worst case (the item is not found) we traverse the entire sequence to look for an item with the given key
- Effective only for dictionaries of
  - small size or
  - when insertions are the most common operations, while searches and removals are rarely performed (e.g., historical record of logins to a workstation)

Hash Table -based Dictionaries

Hash Functions and Hash Tables

- A hash table for a given key type consists of
  - Array (called table) of size N
  - Hash function h

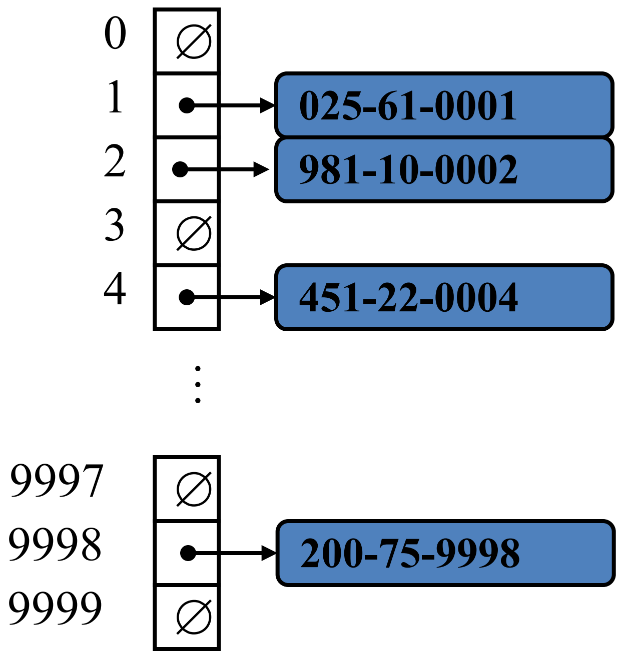
- A hash function h maps keys of a given type to integers in a fixed interval $[0, N - 1]$
  - Ex: $h(x) = x \bmod N$ is a hash function for integer keys
  - The integer $h(x)$ is called the hash value of key x

- When implementing a dictionary with a hash table, the goal is to store item (k, o) at index $i = h(k)$

Example

- We design a hash table for a dictionary storing items (social security number, name)

- Our hash table uses an array of size $N = 10{,}000$ and the hash function $h(x)$ = last four digits of x



Hash Functions

- A hash function is usually specified as the composition of two functions:

  Hash code map
  h_1: keys $\rightarrow$ integers

  Compression map
  h_2: integers $\rightarrow [0, N-1]$

  The hash code map is applied first, and the compression map is applied next on the result

$$h(x) = h_2(h_1(x))$$

- The goal of the hash function is to "disperse" the keys in an apparently random way

Hash Code Maps: keys $\rightarrow$ integers

Memory address

- reinterpret the memory address of the key object as an integer
- default hash code of Java objects
- disadvantage: two key objects with equal value have different hash codes

Integer cast

- reinterpret bits of the key as an integer
- suitable for smaller keys (when number of bits in the key is at most the number of bits in an integer)

Hash Code Maps: keys → integers

Component sum

- suitable for larger keys
- partition bits of the key into components of fixed length and sum the components
- disadvantage: many strings will have the same sum

$$h_1(k) = a_0 + a_1 + a_2 + \ldots + a_{n-1}$$

Polynomial accumulation

- good for strings
- partition bits of the key into components of fixed length and evaluate the polynomial

$$h_1(k) = a_0 + a_1 z + a_2 z^2 + \ldots + a_{n-1} z^{n-1}$$

Compression Maps: integers $\rightarrow [0, N-1]$

- A good hash function guarantees the probability that two different keys have the same hash is 1/N.
- The size N of the hash table is usually chosen to be a prime.
  - The reason involves number theory and is beyond the scope of this course

Division

- $h_2(y) = y \bmod N$
- disadvantage: repeated keys of the form $iN + j$ cause collisions

Multiply, Add and Divide (MAD)

- $h_2(y) = ((ay + b) \bmod p) \bmod N$
- This is a "good" hash function (continued next slide…)

Universal Hashing

- Recall that a good hash function guarantees the probability that two different keys have the same hash is 1/N.

- A family of hash functions is universal if for any $0 \le \boldsymbol{j},\boldsymbol{k} \le \boldsymbol{M}-1$,

$$\boldsymbol{Pr}(\ \boldsymbol{h}(\boldsymbol{j})=\boldsymbol{h}(\boldsymbol{k})\ ) \le 1/\boldsymbol{N}$$

Theorem: The set of all functions, *h*, as defined below, is universal.

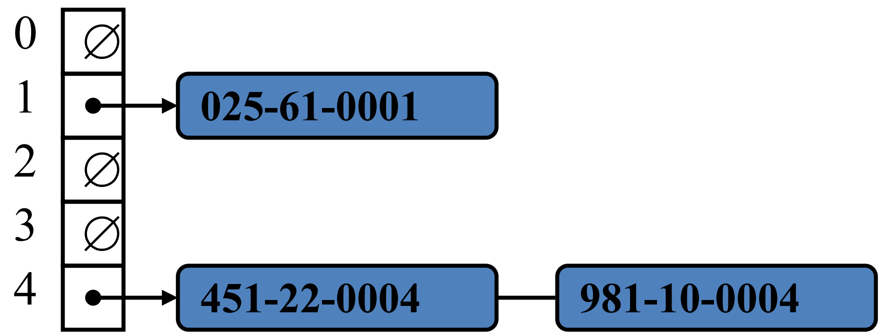
- Choose $\boldsymbol{p}$ as a prime between $\boldsymbol{M}$ and $2\boldsymbol{M}$
- Randomly select $0 < \boldsymbol{a} < \boldsymbol{p}$ and $0 \le \boldsymbol{b} < \boldsymbol{p}$
- $\boldsymbol{a}$ and $\boldsymbol{b}$ are nonnegative integers such that $\boldsymbol{a} \bmod \boldsymbol{N} \neq 0$ (otherwise, every integer would map to the same value $\boldsymbol{b}$)
- Define $\boldsymbol{h}(\boldsymbol{k})=((\boldsymbol{ak}+\boldsymbol{b}) \bmod \boldsymbol{p}) \bmod \boldsymbol{N}$

Collision Handling

Collisions occur when different elements are mapped to the same cell

Chaining

- each cell in the table points to a linked list of elements that map there
- simple, but requires additional memory outside the table



Open Addressing

- the colliding item is placed in a different cell of the table
- no additional memory, but complicates searching/removing
- common types: linear probing, quadratic probing, double hashing

Open Addressing: Linear Probing

- Placing the colliding item in the next (circularly) available table cell

  try $A[(\boldsymbol{h}(\boldsymbol{k}) + \boldsymbol{i}) \bmod N]$ for $\boldsymbol{i} = 0,1,2,\ldots$
- Colliding items cluster together, causing future collisions to cause a longer sequence of probes (searches for next available cell)

- Example:
  - $\boldsymbol{h}(\boldsymbol{x}) = \boldsymbol{x} \bmod 13$
  - Insert keys 18, 41, 22, 44, 59, 32, 31, 73, in this order

0	1	2	3	4	5	6	7	8	9	10	11	12

⇩

		41			18	44	59	32	22	31	73	
0	1	2	3	4	5	6	7	8	9	10	11	12

h(18) = 18 mod 13 = 5
41 mod 13 = 2
22 mod 13 = 9
44 mod 13 = 5
59 mod 13 = 7
32 mod 13 = 6
31 mod 13 = 5
73 mod 13 = 8

Search with Linear Probing

Consider a hash table A that uses linear probing

findElement(k)

- Start at cell $h(k)$
- Check consecutive locations until one of the following occurs
  - An item with key k is found, or
  - An empty cell is found, or
  - N cells have been unsuccessfully probed

```
Algorithm findElement(k)
    i ← h(k)
    p ← 0
    repeat
        c ← A[i]
        if c = ∅
            return NO_SUCH_KEY
        else if c.key () = k
            return c.element()
        else
            i ← (i + 1) mod N
            p ← p + 1
    until p = N
    return NO_SUCH_KEY
```

Updates with Linear Probing

A special object, called ***AVAILABLE***, replaces deleted elements

- removeElement(***k***)
  - Search for an item with key ***k***
  - If it is found, replace it with item ***AVAILABLE*** and return element
  - Else, return ***NO_SUCH_KEY***

- insertItem(***k, o***)
  - Throw an exception if the table is full
  - Start at cell ***h***(***k***)
  - Search consecutive cells until a cell ***i*** is found that is either empty or stores ***AVAILABLE***
  - Store item (***k, o***) in cell ***i***

Open Addressing: Double Hashing

- Use a secondary hash function $\boldsymbol{d}(\boldsymbol{k})$ to place items in first available cell

  try $A[(\boldsymbol{h}(\boldsymbol{k}) + \boldsymbol{i}\boldsymbol{d}(\boldsymbol{k})) \bmod N]$ for $\boldsymbol{i} = 0,1,2,\ldots$
- $\boldsymbol{d}(\boldsymbol{k})$ cannot have zero values

- The table size N must be a prime to allow probing of all the cells

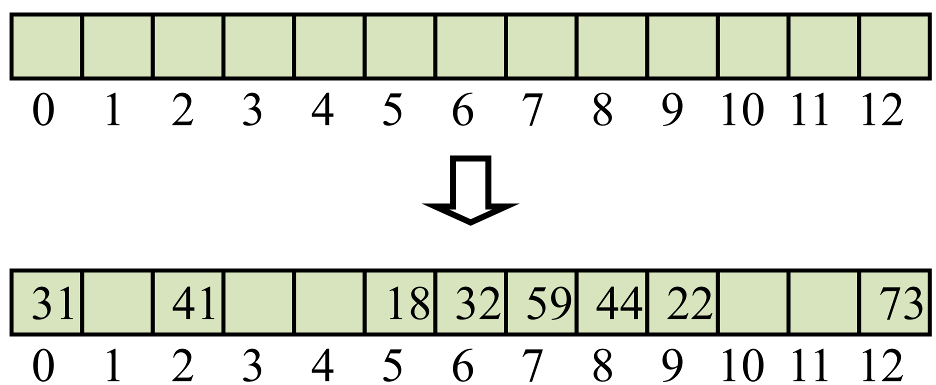
Example of Double Hashing

Consider a hash table storing integer keys that handles collision with double hashing

- $N = 13$
- $h(k) = k \bmod 13$
- $d(k) = 1 + (k \bmod 7)$

Insert keys 18, 41, 22, 44, 59, 32, 31, 73, in this order

k	$h(k)$	$d(k)$	Probes		
18	5	5	5		
41	2	7	2		
22	9	2	9		
44	5	3	5	8	
59	7	4	7		
32	6	5	6		
31	5	4	5	9	0
73	8	4	8	12	



Performance of Hashing

- In the worst case, searches, insertions and removals on a hash table take $O(n)$ time
  - occurs when all inserted keys collide

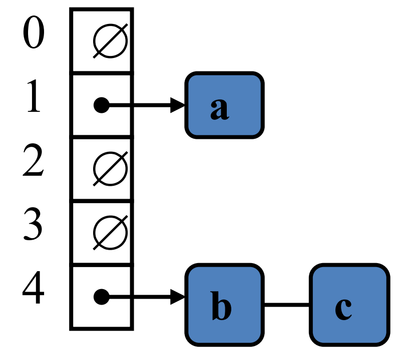
- The **load factor** $\alpha = n/N$ affects the performance of a hash table
  - Assuming that the hash values are like random numbers, it can be shown that the expected number of probes for an insertion with open addressing is $1 / (1 - \alpha)$
  - The expected number of probes for an insertion with chaining is $O(1 + \alpha)$

- The expected running time of all the dictionary ADT operations in a hash table is $O(1)$
- In practice, hashing is very fast provided the load factor is not close to 100%

Chaining vs. Open Addressing

Chaining

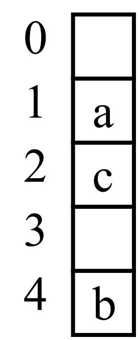
- Less sensitive to hash functions and load factor
- Supports $\alpha > 100\%$



$h(\text{a}) = 1 \quad h(\text{b}) = 4 \quad h(\text{c}) = 4$

Open Addressing

- Requires careful selection of hash function to avoid clustering
- Degrades past $\alpha > 70\%$
- Can't support $\alpha > 100\%$
- Better memory usage



Other

- You are given an array A of integers. Determine the integer that occurs most frequently in A.