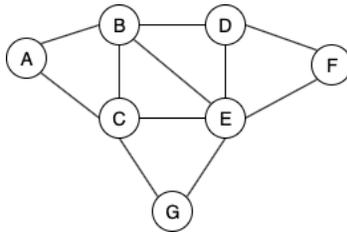


Homework 6 (50 pts)

- (10 points) A company named RT&T has a network of n switching stations connected by m high-speed communication links. Each customer's phone is directly connected to one station in his or her area. The engineers of RT&T have developed a prototype video-phone system that allows two customers to see each other during a phone call. In order to have acceptable image quality, however, the number of links used to transmit video signals between the two parties cannot exceed 4. Suppose that RT&T network is represented by a graph. Design and give the **pseudo-code** for an efficient algorithm that computes, for each station, the set of stations it can reach using no more than 4 links. Analyze its running time.
- (10 points) An Eulerian cycle of a directed graph G with n vertices and m edges is a cycle that traverses each edge of G exactly once according to its direction. Such a cycle always exists if the in-degree is equal to the out-degree for each vertex in G . Describe a $O(n + m)$ time algorithm for finding an Euler cycle of such a graph G . Analyze its running time.
- (10 points) Consider the graph G depicted below, and perform the following graph search algorithms. Whenever faced with a decision of which vertex to pick from a set of vertices, **pick the vertex whose label occurs earliest in the alphabet**.
 - Trace the execution of BFS beginning at vertex A , labeling each edge as a discovery or cross edge.
 - Trace the execution of DFS beginning at vertex A , labeling each edge as a discovery or back edge.



- (10 points) Specify which graph searching algorithm, DFS, or BFS, or both, can be used to efficiently solve the following tasks on an undirected graph. In each case, describe in a few sentences how the task is accomplished via DFS or BFS using only the discovery-edges or back/cross-edges (depending on the searching algorithm).
 - Find a spanning tree.
 - Determine if the graph is acyclic.
 - Find a path from a vertex u to a vertex v .
 - Find a shortest path from a vertex u to a vertex v .
 - Find the connected components of the graph.
- (5 points) A graph is **triconnected** if one has to remove at least 3 vertices from the graph to disconnect it. Construct examples of the following graphs or explain why it cannot be done. Assume the graph is undirected.
 - A triconnected graph with exactly 5 vertices and 8 edges.
 - A triconnected graph with exactly 5 vertices and 6 edges.
 - A triconnected graph with exactly 8 vertices and 14 edges.

6. (5 points) Bob loves foreign languages and wants to plan his course schedule to take the following nine language courses: LA15, LA16, LA22, LA31, LA32, LA126, LA127, LA141, and LA169. The course prerequisites are:

- LA15: (none)
- LA16: LA15
- LA22: (none)
- LA31: LA15
- LA32: LA16, LA31
- LA126: LA22, LA32
- LA127: LA16
- LA141: LA22, LA16
- LA169: LA32

Find a sequence of courses that allows Bob to satisfy all the prerequisites.