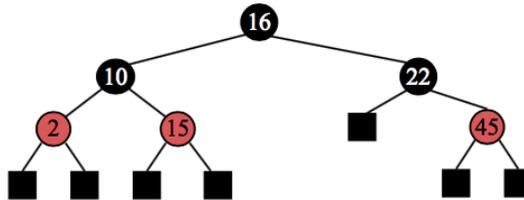


## Design and Analysis of Algorithms: Homework 3

1. (a) (4 points) Insert into an initially empty binary search tree items with the following keys (in this order): 30, 40, 23, 58, 48, 26, 11, 13. Draw the tree after all insertions. Include a few intermediate stages.
- (b) (4 points) Remove from the binary search tree in Figure 3.7(a) the following keys (in this order): 32, 65, 76, 88, 97. Draw the tree after **each** removal.
- (c) (2 points) A different binary search tree results when we try to insert the same sequence into an empty BST in a different order. Give an example of this with at least 5 elements and show the two different binary search trees that result.
2. (a) (5 points) Let  $T$  be a binary search tree, and let  $x$  be a key. Give an efficient algorithm for finding the smallest key  $y$  in  $T$  such that  $y > x$ . Note that  $x$  may or may not be in  $T$ . Explain why your algorithm has the running time it does.
- (b) (5 points) Give the **pseudocode** for a **nonrecursive** linear-time algorithm that prints out the keys from a binary search tree in order. You can assume the existence of a  $O(1)$ -time  $print(key)$  function.
3. (a) (5 points) Consider the following sequence of keys: (18, 30, 50, 12, 1). Insert the items with this set of keys in the order given into the red-black tree in the figure below. Draw the tree after **each** insertion.



- (b) (5 points) Design and give the **pseudocode** for an  $O(\log n)$  algorithm that determines whether a red-black tree with  $n$  keys stores any keys within a certain (closed) interval. That is, the input to the algorithm is a red-black tree  $T$  and two keys,  $l$  and  $r$ , where  $l \leq r$ . If  $T$  has at least one key  $k$  such that  $l \leq k \leq r$ , then the algorithm returns true, otherwise it returns false. *Hint:* You can use the `TreeSearch` algorithm (page 146) as a subroutine.
4. (a) (5 points) Draw the merge-sort tree for an execution of the merge-sort algorithm on the input sequence: (2, 5, 16, 4, 10, 23, 39, 18, 26, 15) (like in Figure 4.2).
- (b) (5 points) Draw the quick-sort tree for an execution of the quick-sort algorithm on the input sequence from part (a) (like in Figure 4.12). Use the last element as the pivot.
- (c) (5 points) Draw the quick-sort tree for an execution of the quick-sort algorithm on the input sequence from part (a) (like in Figure 4.12). Use the element at rank  $\lfloor \frac{n}{2} \rfloor$  as the pivot.
- (d) (2 points) What is the running time of the version of quick-sort in part (c) on a sequence that is already sorted? Explain.
5. (a) (5 points) Suppose we are given a sequence  $S$  of  $n$  elements, each of which is colored red or blue. Assuming  $S$  is represented by an array, give a linear-time **in-place** algorithm for ordering  $S$  so that all the blue elements are listed before all the red elements. What is the running time of your method?

- (b) (5 points) Let  $A$  and  $B$  be two sequences of  $n$  integers each. Give an integer  $m$ , describe an  $O(n \log n)$  time algorithm for determining if there is an integer  $a$  in  $A$  and an integer  $b$  in  $B$  such that  $m = a + b$ .
6. (a) (5 points) Suppose we are given a sequence  $S$  of  $n$  elements, each of which is an integer in the range  $[0, n^2 - 1]$ . Describe a simple method for sorting  $S$  in  $O(n)$  time. [*Hint*: think of alternative ways of viewing the elements].
- (b) (3 points) Does the running time of radix-sort depend on the order of keys in the input? Explain.