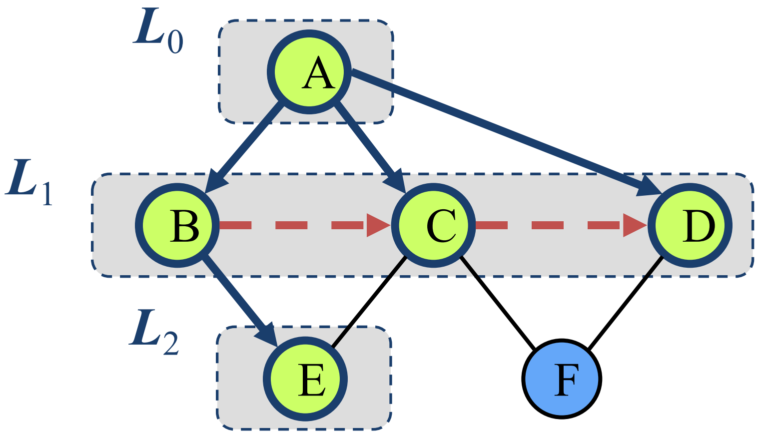
# Breadth-First Search



# Outline and Reading

Breadth-first search (6.3.3)

- Algorithm
- Example
- Properties
- Analysis
- Applications

DFS vs. BFS (6.3.3)

- Comparison of applications
- Comparison of edge labels

# Breadth-First Search

- Breadth-first search (BFS) is a general technique for traversing a graph. A BFS traversal of a graph G
  - visits all the vertices and edges of G
  - determines whether G is connected
  - computes the connected components of G
  - computes a spanning forest of G

- BFS on a graph with $n$ vertices and $m$ edges takes $O(n + m)$ time

- BFS can be further extended to solve other graph problems
  - find and report a path with the minimum number of edges between two given vertices
  - find a simple cycle, if there is one

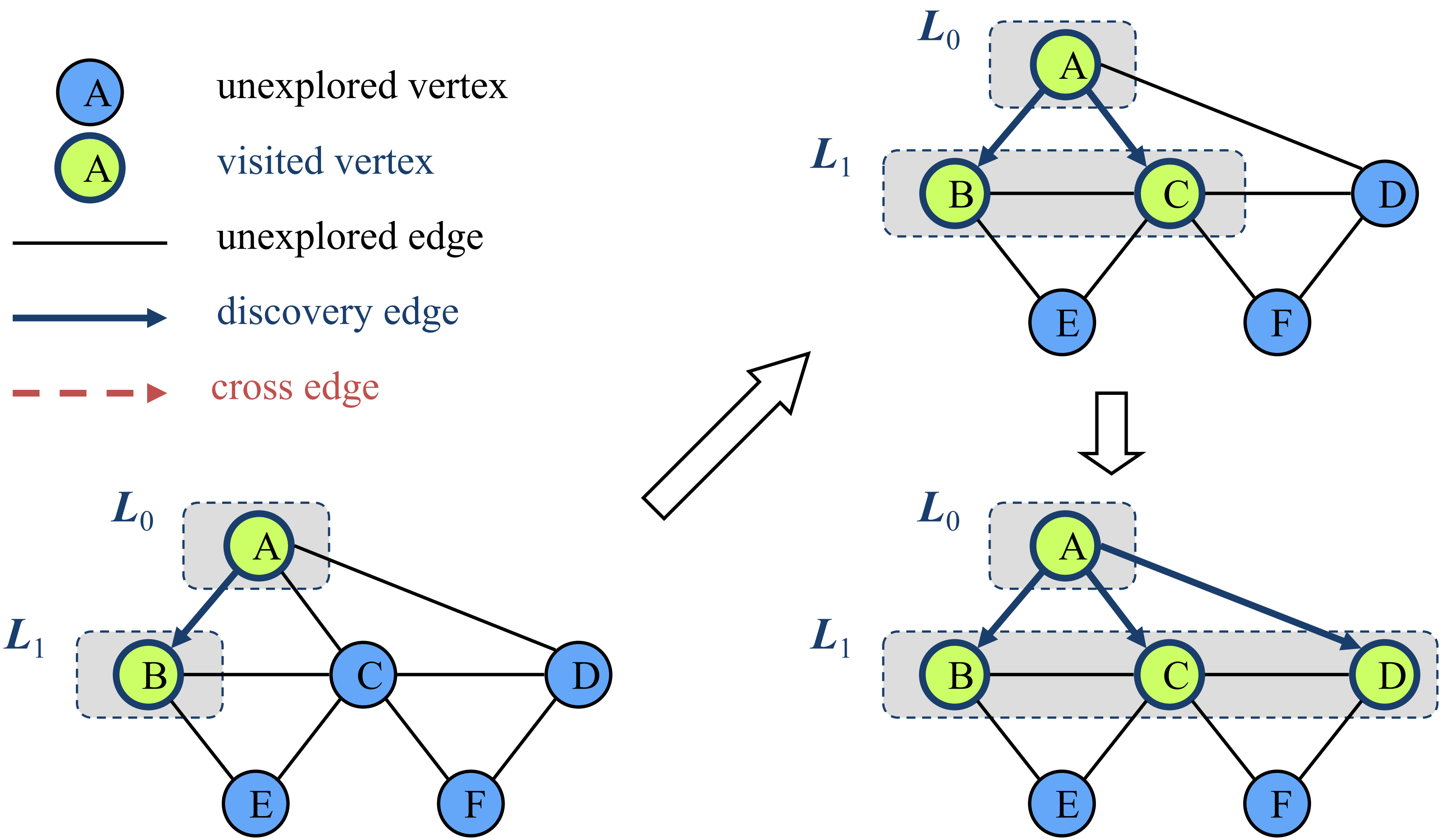
# BFS Algorithm

The algorithm uses a mechanism for setting and getting "labels" of vertices and edges.

```
Algorithm BFS(G)
   Input graph G
   Output labeling of the edges
      and partition of the
      vertices of G
  for all u ∈ G.vertices()
   setLabel(u, UNEXPLORED)
  for all e ∈ G.edges()
   setLabel(e, UNEXPLORED)
  for all v ∈ G.vertices()
   if getLabel(v) = UNEXPLORED
      BFS(G, v)
```

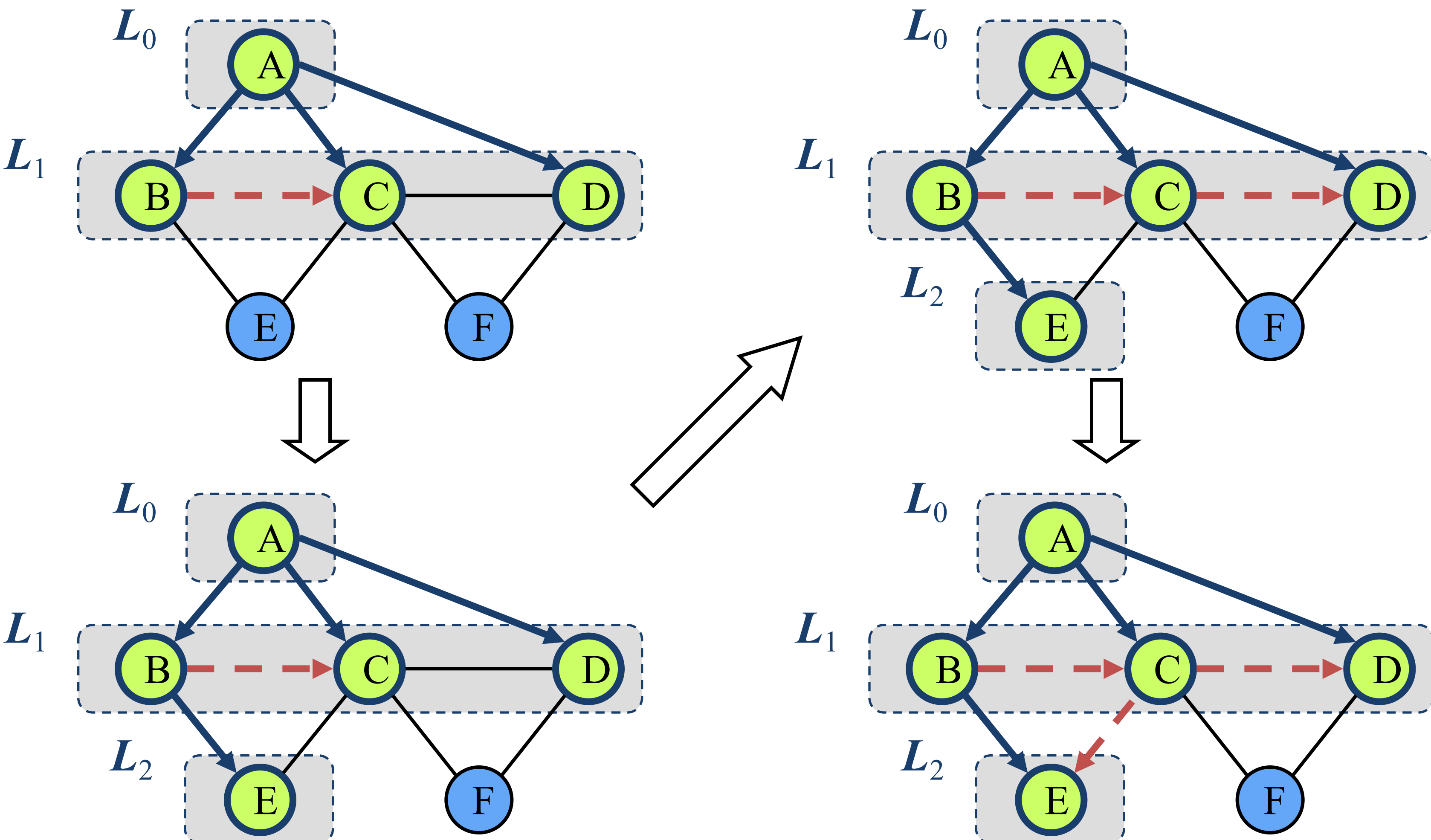
```
Algorithm BFS(G, s)
  L0 ← new empty sequence
  L0.insertLast(s)
  setLabel(s, VISITED)
  i ← 0
  while ¬Li.isEmpty()
    Li+1 ← new empty sequence
    for all v ∈ Li.elements()
      for all e ∈ G.incidentEdges(v)
        if getLabel(e) = UNEXPLORED
          w ← opposite(v,e)
          if getLabel(w) = UNEXPLORED
            setLabel(e, DISCOVERY)
            setLabel(w, VISITED)
            Li+1.insertLast(w)
          else
            setLabel(e, CROSS)
    i ← i +1
```

# Example

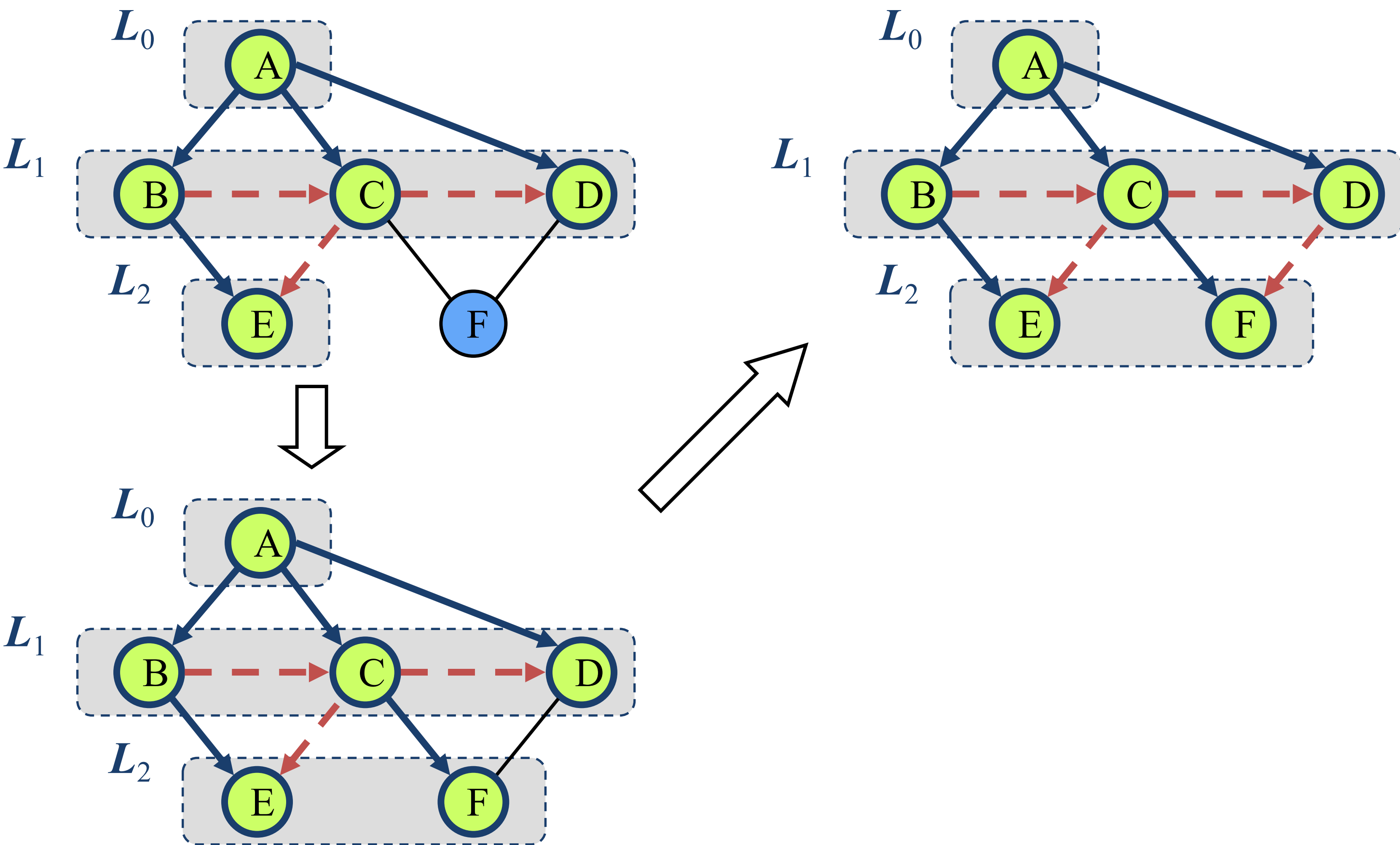


A
unexplored vertex
A
visited vertex
unexplored edge
discovery edge
cross edge
$L_0$
$L_1$
A
B
C
D
E
F
$L_0$
$L_1$
A
B
C
D
E
F
$L_0$
$L_1$
A
B
C
D
E
F

# Example (cont.)



# Example (cont.)



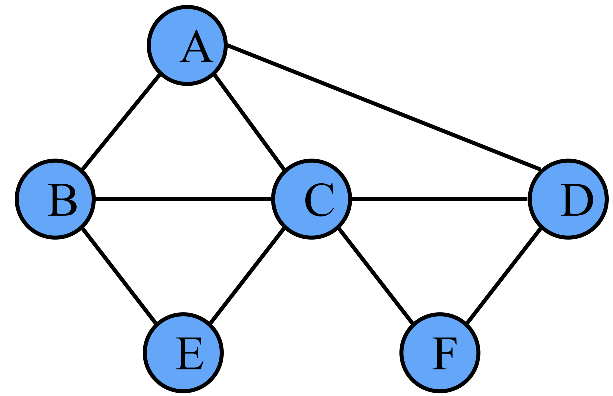
# Properties

### Notation

$G_s$: connected component of $s$

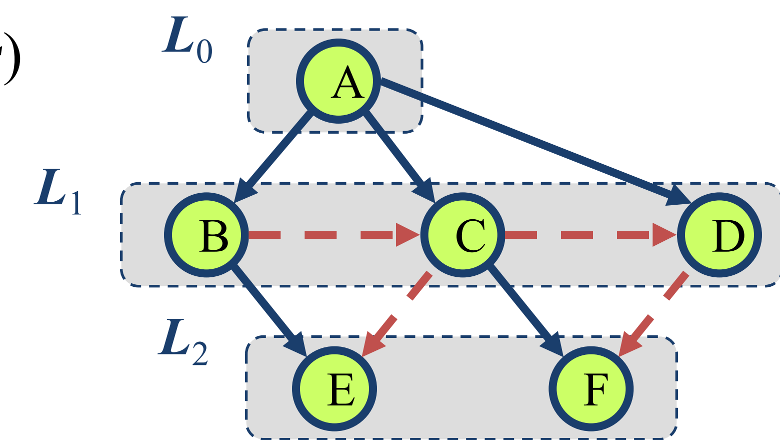
### Property 1

$BFS(G, s)$ visits all the vertices and edges of $G_s$




### Property 2

The discovery edges labeled by $BFS(G, s)$ form a spanning tree $T_s$ of $G_s$




### Property 3

For each vertex $v$ in $L_i$

- The path of $T_s$ from $s$ to $v$ has $i$ edges
- Every path from $s$ to $v$ in $G_s$ has at least $i$ edges

# Analysis

- Setting/getting a vertex/edge label takes $O(1)$ time
- Each vertex is labeled twice
  - once as UNEXPLORED
  - once as VISITED
- Each edge is labeled twice
  - once as UNEXPLORED
  - once as DISCOVERY or CROSS
- Each vertex is inserted once into a sequence $L_i$
- Method incidentEdges is called once for each vertex
- BFS runs in $O(n + m)$ time provided the graph is represented by the adjacency list structure
  - Recall that $\Sigma_v \deg(v) = 2m$

# Applications

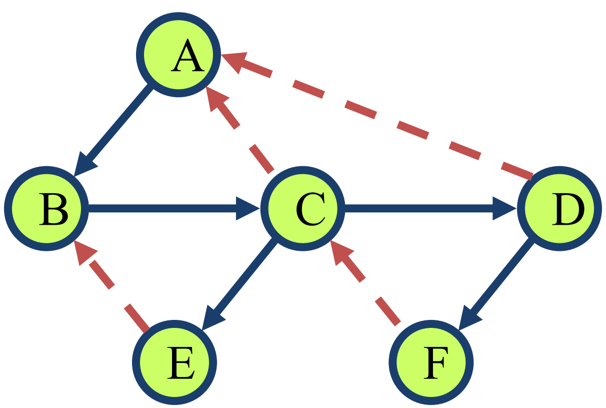
Using the template method pattern, we can specialize the BFS traversal of a graph $G$ to solve the following problems in $O(n + m)$ time:

- Compute the connected components of $G$
- Compute a spanning forest of $G$
- Find a simple cycle in $G$, or report that $G$ is a forest
- Given two vertices of $G$, find a path in $G$ between them with the minimum number of edges, or report that no such path exists

# DFS vs. BFS

**Back edge** ***(v,w)***

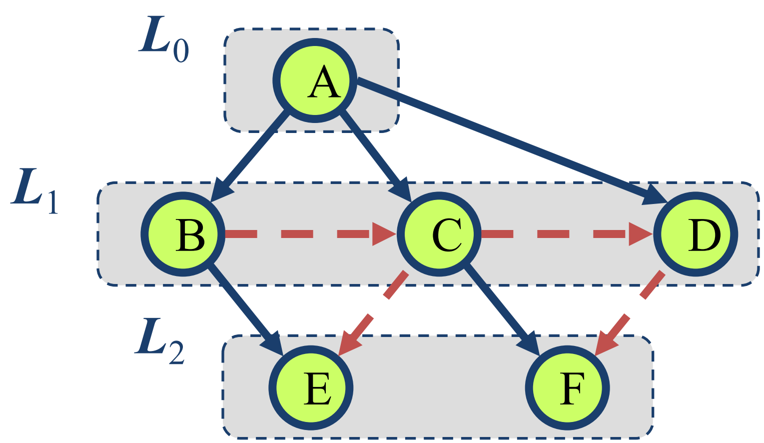
- ***w*** is an ancestor of ***v*** in the tree of discovery edges




DFS

**Cross edge** ***(v,w)***

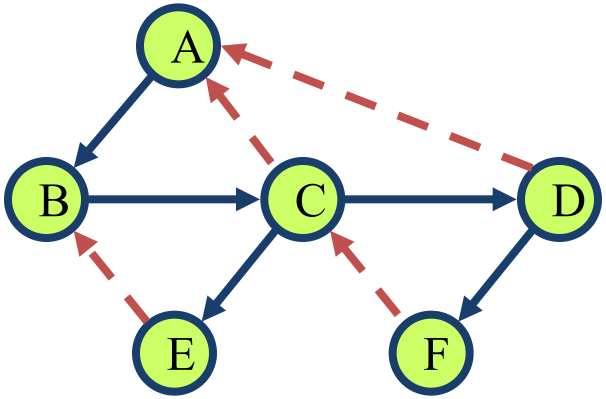
- ***w*** is in the same level as ***v*** or in the next level in the tree of discovery edges



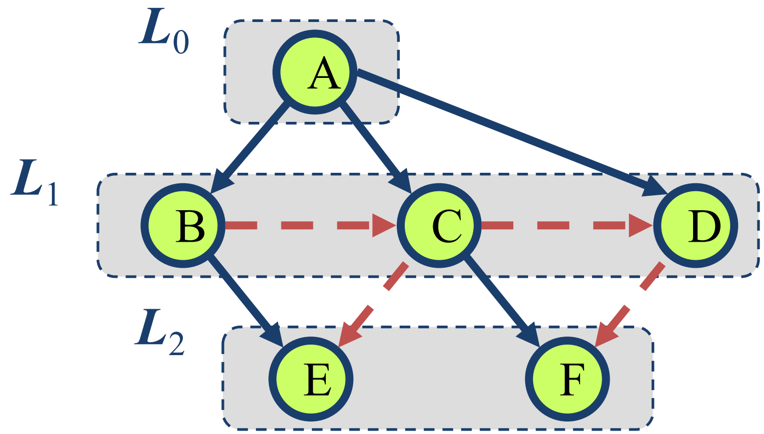

BFS

# DFS vs. BFS

| Applications | DFS | BFS |
|---|---|---|
| Spanning forest, connected components, paths, cycles | √ | √ |
| Shortest paths | | √ |
| Biconnected components | √ | |




DFS




BFS