Sets

Set ADT

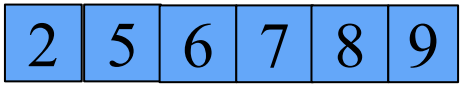
- A collection of unordered distinct objects
  - there is no inherent ordering of elements in a set, but keeping the elements sorted can lead to more efficient set operations

- Main operations
  - union(B): executes $A \leftarrow A \cup B$
  - intersect(B): executes $A \leftarrow A \cap B$
  - subtract(B): executes $A \leftarrow A - B$
  - implemented using a generic version of the merge algorithm

- Running time of an operation should be at most $O(n_A+n_B)$

Storing a Set in a List

- We can implement a set with a list
- Elements are sorted according to some canonical ordering
- Space used is $O(n)$



Generic Merging

- Generalized merge of two sorted lists *A* and *B*
- Auxiliary methods aIsLess, bIsLess, bothAreEqual decide whether to add the element to list *S* based on what main operation is performed

```
Algorithm genericMerge(A, B)
    S ← empty sequence
    while ¬A.isEmpty() ∧ ¬B.isEmpty()
        a ← A.first().element(); b ← B.first().element()
        if a < b
            aIsLess(a, S); A.remove(A.first())
        else if b < a
            bIsLess(b, S); B.remove(B.first())
        else { b = a }
            bothAreEqual(a, b, S)
            A.remove(A.first()); B.remove(B.first())
    while ¬A.isEmpty()
        aIsLess(a, S); A.remove(A.first())
    while ¬B.isEmpty()
        bIsLess(b, S); B.remove(B.first())
    return S
```

Example: Union

- if $a < b$, copy a to output sequence and go to next element of A
- if $a = b$, copy a to output sequence and go to next element of A and B
- if $a > b$, copy b to output sequence and go to next element of B

$S = A \cup B$

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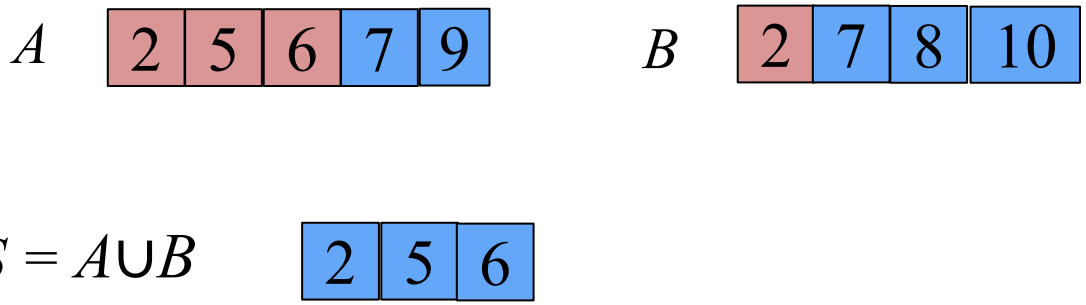

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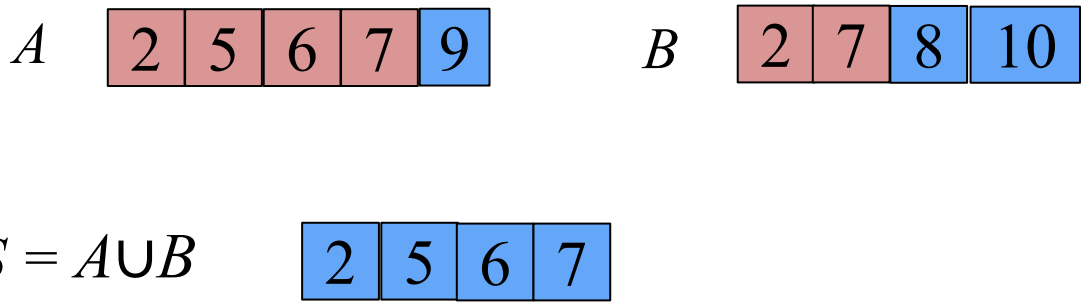

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Using Generic Merge for Set Operations

- Any of the set operations can be implemented using a generic merge

- For example:
  - intersection: only copy elements that are duplicated in both lists
  - subtraction: only copy elements from *A* that are not equal to those in *B*

- All methods run in linear time.