Design and Analysis of Algorithms
Homework 5

Clearly number your solution to each problem. Staple your solutions and bring them to class on the due date. Express your algorithms in pseudo-code when directed. Always provide justification for your answer when asked to give the running time of an algorithm. Be brief and concise, and draw pictures where appropriate.

1. (10pts) Let $S = \{a, b, c, d, e, f, g\}$ be a collection of items with weight-benefit values as follows: $a(3, 12)$, $b(6, 12)$, $c(6, 9)$, $d(1, 5)$, $e(2, 5)$, $f(10, 10)$, $g(3, 9)$. For example, item a weighs 3 lbs and is worth a total of \$12. What is an optimal solution to the fractional knapsack problem for S assuming we have a knapsack that can hold a total of 11 lbs? Show your work.

2. (10pts) Suppose we are given a set of tasks specified by pairs of start times and finish times as $T = \{(1, 2), (1, 3), (1, 4), (2, 5), (3, 7), (4, 9), (5, 6), (6, 8), (7, 9), (7, 10)\}$. Solve the task scheduling problem for these tasks.

3. (10pts) Characterize each of the following recurrence equations using the master method (assuming that $T(n) = c$ for $n < d$, for constants $c > 0$ and $d \geq 1$).
   (a) $T(n) = 2T(n/2) + \log n$
   (b) $T(n) = 8T(n/2) + n^2$
   (c) $T(n) = 7T(n/3) + n$
   (d) $T(n) = 4T(n/2) + n^2$
   (e) $T(n) = 3T(n/2) + n^2$

4. (10pts) What is the best way to multiply a chain of matrices with dimensions that are 10×5, 5×2, 2×20, 20×12, 12×4, and 4×60? Show your work.

5. (10pts) Consider the single machine scheduling problem where we are given a set T of tasks specified by their start times and finish times, as in the task scheduling problem, except now we have only one machine and we wish to maximize the number of tasks that this single machine performs.
   (a) Design a greedy algorithm for this single machine scheduling problem. What is the running time of your algorithm?
   (b) Show that your algorithm is correct.